2.3 Multiplying Fractions

Essential Question What does it mean to multiply fractions?

1 **EXAMPLE: Multiplying Fractions**

A bottle of water is $\frac{4}{5}$ full. You drink $\frac{2}{3}$ of the water. How much do you drink?

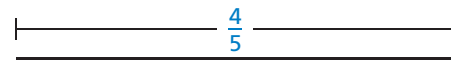**THINK ABOUT THE QUESTION:** To help you think about this question, rewrite the question.

Words: What is $\frac{2}{3}$ of $\frac{4}{5}$?

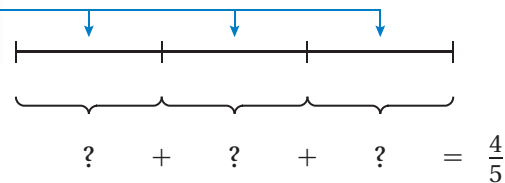**Numbers:** $\frac{2}{3} \times \frac{4}{5} = ?$

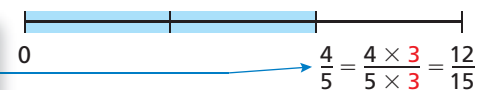Here is one way to get the answer.

- **Draw** a length of $\frac{4}{5}$.

$$\frac{4}{5}$$

Because you want to find $\frac{2}{3}$ of the length, divide it into 3 equal sections.

$$? \quad + \quad ? \quad + \quad ? \quad = \quad \frac{4}{5}$$

Now, you need to think of a way to divide $\frac{4}{5}$ into three equal parts.

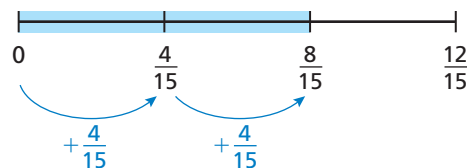- **Rewrite** $\frac{4}{5}$ as a fraction whose numerator is divisible by 3.

Because the length is divided into 3 equal sections, multiply the numerator and denominator by 3.

$$0 \qquad \frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

In this form, you see that $\frac{12}{15}$ can be divided into three equal parts of $\frac{4}{15}$.

- Each part is $\frac{4}{15}$ of the water and you drank two of them. Written as multiplication, you have

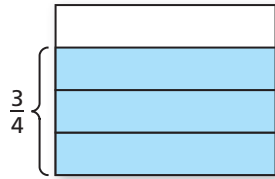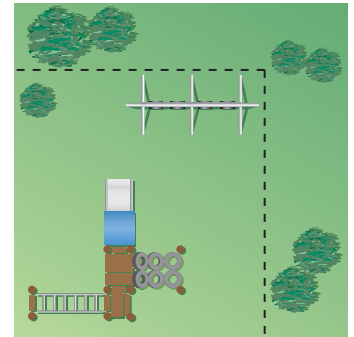$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}.$$

$$0 \qquad \frac{4}{15} \qquad \frac{8}{15} \qquad \frac{12}{15}$$

$$+\frac{4}{15} \qquad +\frac{4}{15}$$
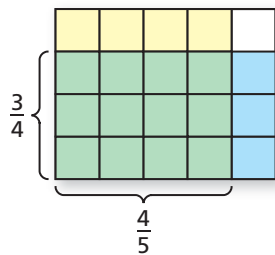
So, you drank $\frac{8}{15}$ of the water.

EXAMPLE: Multiplying Fractions

A park has a playground that is $\frac{3}{4}$ of its width and $\frac{4}{5}$ of its length.

What fraction of the park is covered by the playground?

$\frac{3}{4}\Big\{$

Fold a piece of paper horizontally into fourths and shade three of the fourths to represent $\frac{3}{4}$.

$\frac{3}{4}\Big\{$

$\frac{4}{5}$

Fold the paper vertically into fifths and shade $\frac{4}{5}$ of the paper another color.

Count the total number of squares. This number is the denominator. The numerator is the number of squares shaded with both colors.

$$\frac{3}{4} \times \frac{4}{5} = \frac{12}{20} = \frac{3}{5}. \text{ So, } \frac{3}{5} \text{ of the park is covered by the playground.}$$

Inductive Reasoning

Work with a partner. Complete the table using a model or paper folding.

Exercise	Verbal Expression	Answer
3. $\frac{2}{3} \times \frac{4}{5}$	$\frac{2}{3}$ of $\frac{4}{5}$	$\frac{8}{15}$
4. $\frac{3}{4} \times \frac{4}{5}$	$\frac{3}{4}$ of $\frac{4}{5}$	$\frac{3}{5}$
5. $\frac{2}{3} \times \frac{5}{6}$		
6. $\frac{1}{6} \times \frac{1}{4}$		
7. $\frac{2}{5} \times \frac{1}{2}$		
8. $\frac{5}{8} \times \frac{4}{5}$		

What Is Your Answer?

9. IN YOUR OWN WORDS What does it mean to multiply fractions?

10. Write a general rule for multiplying fractions.

Practice Use what you learned about multiplying fractions to complete Exercises 5–12 on page 60.

2.3 Lesson

🔑 Key Idea

Multiplying Fractions

Words Multiply the numerators and multiply the denominators.

Numbers $\dfrac{3}{7} \times \dfrac{1}{2} = \dfrac{3 \times 1}{7 \times 2} = \dfrac{3}{14}$

Algebra $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{a \cdot c}{b \cdot d}$, where $b, d \neq 0$

EXAMPLE 1 **Multiplying Fractions**

Find $\dfrac{1}{5} \times \dfrac{1}{3}$.

$\dfrac{1}{5} \times \dfrac{1}{3} = \dfrac{1 \times 1}{5 \times 3}$ ← Multiply the numerators.
← Multiply the denominators.

$= \dfrac{1}{15}$ Simplify.

EXAMPLE 2 **Multiplying Fractions with Common Factors**

Find $\dfrac{8}{9} \times \dfrac{3}{4}$.   **Estimate** $1 \times \dfrac{1}{2} = \dfrac{1}{2}$

Study Tip

When the numerator of one fraction is the same as the denominator of another fraction, you can use mental math to multiply. For example, $\dfrac{4}{5} \times \dfrac{5}{9} = \dfrac{4}{9}$ because you can divide out the common factor 5.

$\dfrac{8}{9} \times \dfrac{3}{4} = \dfrac{8 \times 3}{9 \times 4}$ ← Multiply the numerators.
← Multiply the denominators.

$= \dfrac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{1}{\cancel{4}}}$ Divide out common factors.

$= \dfrac{2}{3}$ Simplify.

∴ So, the product is $\dfrac{2}{3}$.   **Reasonable?** $\dfrac{2}{3} \approx \dfrac{1}{2}$ ✓

On Your Own

Now You're Ready
Exercises 5–20

Multiply. Write the answer in simplest form.

1. $\dfrac{1}{2} \times \dfrac{5}{6}$ **2.** $\dfrac{7}{8} \times \dfrac{1}{4}$ **3.** $\dfrac{3}{7} \times \dfrac{2}{3}$ **4.** $\dfrac{4}{9} \times \dfrac{3}{10}$

EXAMPLE 3 **Standardized Test Practice**

What is the value of $p \cdot \dfrac{7}{8} - q$ when $p = \dfrac{4}{5}$ and $q = \dfrac{1}{4}$?

(A) $\dfrac{1}{4}$ (B) $\dfrac{9}{20}$ (C) $\dfrac{1}{2}$ (D) 1

$p \cdot \dfrac{7}{8} - q = \dfrac{4}{5} \cdot \dfrac{7}{8} - \dfrac{1}{4}$ Substitute $\dfrac{4}{5}$ for p and $\dfrac{1}{4}$ for q.

$= \dfrac{\overset{1}{\cancel{4}} \cdot 7}{5 \cdot \underset{2}{\cancel{8}}} - \dfrac{1}{4}$ Multiply. Divide out the common factor 4.
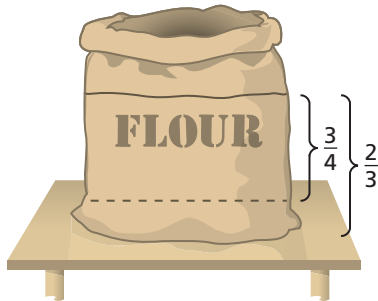
$= \dfrac{7}{10} - \dfrac{1}{4}$ Simplify.

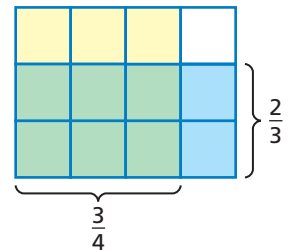$= \dfrac{14}{20} - \dfrac{5}{20} = \dfrac{9}{20}$ Subtract.

⋮ The correct answer is (B).

EXAMPLE 4 **Real-Life Application**

You have $\dfrac{2}{3}$ of a bag of flour. You use $\dfrac{3}{4}$ of the flour to make empanada dough. How much of the entire bag do you use to make the dough?

Method 1: Use a model. Six of the 12 squares have both types of shading.

⋮ So, you use $\dfrac{6}{12} = \dfrac{1}{2}$ of the entire bag.

Method 2: To find $\dfrac{3}{4}$ of $\dfrac{2}{3}$, multiply.

$\dfrac{3}{4} \times \dfrac{2}{3} = \dfrac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{1}{\cancel{3}}}$ Multiply the numerators and the denominators.
Divide out common factors.

$= \dfrac{1}{2}$ Simplify.

⋮ So, you use $\dfrac{1}{2}$ of the entire bag.

On Your Own

5. Evaluate $a + b \cdot \dfrac{1}{12}$ when $a = \dfrac{5}{6}$ and $b = \dfrac{2}{3}$.

6. WHAT IF? In Example 4, you use $\dfrac{1}{4}$ of the flour to make the dough. How much of the entire bag do you use to make the dough?
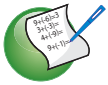
✔️ Vocabulary and Concept Check

1. **WRITING** Explain how to multiply two fractions.

2. **OPEN-ENDED** Give three different sets of two fractions each having the same product.

3. **REASONING** Name the missing denominator.

$$\frac{3}{7} \times \frac{1}{\boxed{}} = \frac{3}{28}$$

4. **NUMBER SENSE** Is $\frac{2}{3} \times \frac{5}{8}$ the same as $\frac{5}{8} \times \frac{2}{3}$? Explain.

✏️ Practice and Problem Solving

Multiply. Write the answer in simplest form.

① ② **5.** $\frac{1}{7} \times \frac{2}{3}$ **6.** $\frac{5}{8} \times \frac{1}{2}$ **7.** $\frac{1}{4} \times \frac{2}{5}$ **8.** $\frac{3}{7} \times \frac{1}{4}$

9. $\frac{2}{3} \times \frac{4}{5}$ **10.** $\frac{5}{7} \times \frac{7}{8}$ **11.** $\frac{3}{8} \times \frac{1}{9}$ **12.** $\frac{5}{6} \times \frac{2}{5}$

13. $\frac{5}{12} \times 10$ **14.** $6 \times \frac{7}{8}$ **15.** $\frac{3}{4} \times \frac{8}{15}$ **16.** $\frac{4}{9} \times \frac{4}{5}$

17. $\frac{3}{7} \times \frac{3}{7}$ **18.** $\frac{5}{6} \times \frac{2}{9}$ **19.** $\frac{13}{18} \times \frac{6}{7}$ **20.** $\frac{7}{9} \times \frac{21}{10}$

21. **ERROR ANALYSIS** Describe and correct the error in finding the product.

❌ $\dfrac{2}{5} \times \dfrac{3}{10} = \dfrac{4}{10} \times \dfrac{3}{10} = \dfrac{4 \times 3}{10} = \dfrac{12}{10} = 1\dfrac{1}{5}$

22. **AQUARIUM** In an aquarium, $\frac{2}{5}$ of the fish are surgeonfish. Of these, $\frac{3}{4}$ are yellow tangs. What fraction of all fish in the aquarium are yellow tangs?

23. **JUMP ROPE** You exercise for $\frac{3}{4}$ of an hour. You jump rope for $\frac{1}{3}$ of that time. What fraction of the hour do you spend jumping rope?

ALGEBRA Evaluate the expression when $a = \dfrac{3}{4}, b = \dfrac{1}{6},$ and $c = \dfrac{2}{5}$.

③ **24.** $a \cdot \dfrac{5}{12}$ **25.** $\dfrac{4}{7} \cdot b$ **26.** $a \cdot b$ **27.** $c \cdot a$

28. $\dfrac{5}{9}a + \dfrac{1}{9}$ **29.** $\dfrac{14}{15} - \dfrac{7}{12}c$ **30.** $bc + \dfrac{2}{3}$ **31.** $ab + c$

Multiply. Write the answer in simplest form.

32. $\dfrac{1}{2} \times \dfrac{3}{5} \times \dfrac{4}{9}$

33. $\dfrac{3}{4} \times \dfrac{5}{8} \times \dfrac{6}{25}$

34. $\dfrac{4}{7} \times \dfrac{2}{3} \times \dfrac{9}{16}$

35. $\dfrac{5}{6} \times \dfrac{4}{15} \times \dfrac{7}{10}$

36. $\left(\dfrac{9}{10}\right)^2$

37. $\left(\dfrac{3}{5}\right)^3$

38. $\left(\dfrac{4}{5}\right)^2 \times \left(\dfrac{3}{4}\right)^2$

39. $\left(\dfrac{5}{6}\right)^2 \times \left(\dfrac{3}{7}\right)^2$

**Without finding the product, copy and complete the statement using <, >, or =. Explain your reasoning.**

40. $\dfrac{4}{7}$ ▢ $\left(\dfrac{9}{10} \times \dfrac{4}{7}\right)$

41. $\left(\dfrac{5}{8} \times \dfrac{22}{15}\right)$ ▢ $\dfrac{5}{8}$

42. $\dfrac{5}{6}$ ▢ $\left(\dfrac{5}{6} \times \dfrac{7}{7}\right)$

43. OPEN-ENDED Find a fraction that when multiplied by $\dfrac{1}{2}$ is less than $\dfrac{1}{4}$.

44. DISTANCES You are in a bike race. When you get to the first checkpoint, you are $\dfrac{2}{5}$ the distance to the second checkpoint. When you get to the second checkpoint, you are $\dfrac{1}{4}$ the distance to the finish. What is the distance from the start to the first checkpoint?

Start   Checkpoint 1   Checkpoint 2   Finish   FINISH

40 miles
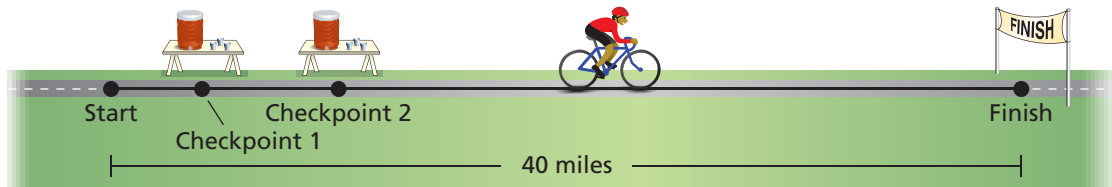
45. PETS You ask 150 people about their pets. The results show that $\dfrac{9}{25}$ of the people own a dog. Of the people that own a dog, $\dfrac{1}{6}$ of them also own a cat.

a. What fraction of the people own a dog and a cat?

b. **Reasoning** How many people own a dog, but not a cat? Explain.

Fair Game Review What you learned in previous grades & lessons

Write the mixed number as an improper fraction.   *(Skills Review Handbook)*

46. $9\dfrac{1}{3}$

47. $4\dfrac{3}{8}$

48. $7\dfrac{3}{4}$

49. $3\dfrac{5}{6}$

50. MULTIPLE CHOICE A science experiment calls for $\dfrac{3}{4}$ cup of baking powder. You have $\dfrac{1}{3}$ cup of baking powder. How much more baking powder do you need? *(Skills Review Handbook)*

(A) $\dfrac{1}{4}$ cup

(B) $\dfrac{5}{12}$ cup

(C) $\dfrac{4}{7}$ cup

(D) $1\dfrac{1}{12}$ cups