

2.5 Dividing Fractions

Essential Question How do you divide by a fraction?

1 ACTIVITY: Dividing by a Fraction

Work with a partner.

- Describe the pattern of the **blue numbers**.
- Describe the pattern of the **red numbers**.
Use the pattern to complete the table.
- The division $8 \div \frac{1}{2}$ can be read as “How many halves are in 8?” Use the completed table to answer this question. Then draw a model that shows your answer.
- Use the pattern in the table to complete the following.

$$8 \div \frac{1}{2} = 16 = 8 \times \frac{2}{1}$$

Invert $\frac{1}{2}$ and multiply.

$$8 \div \frac{1}{4} = 32 = \text{[yellow box]}$$

[yellow box]

$$8 \div \frac{1}{8} = 64 = \text{[yellow box]}$$

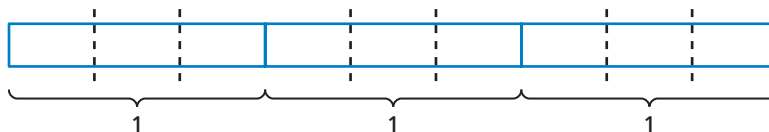
[yellow box]

$8 \div 16$	$\frac{1}{2}$
$8 \div 8$	1
$8 \div 4$	2
$8 \div 2$	4
$8 \div 1$	8
$8 \div \frac{1}{2}$	
$8 \div \frac{1}{4}$	
$8 \div \frac{1}{8}$	

2 ACTIVITY: Dividing by a Fraction

Work with a partner.

- Draw a model for $3 \div \frac{2}{3}$. Use the model to answer the question “How many two-thirds are in 3?”



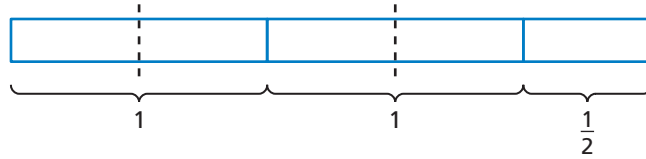
- Complete the table in two ways. First use the model. Then use the “invert and multiply” rule that you found in Activity 1. Compare your answers.

$3 \div \frac{2}{3}$	
$6 \div \frac{2}{3}$	
$9 \div \frac{2}{3}$	
$12 \div \frac{2}{3}$	

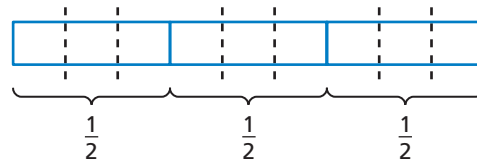
3 ACTIVITY: Dividing by a Fraction

Work with a partner. Write the division problem and answer it using a model.

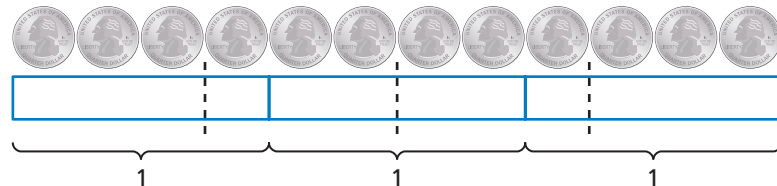
- a. How many halves are in five halves?



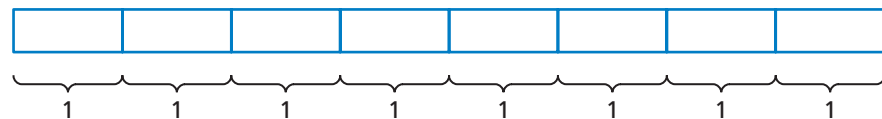
- b. How many sixths are in three halves?



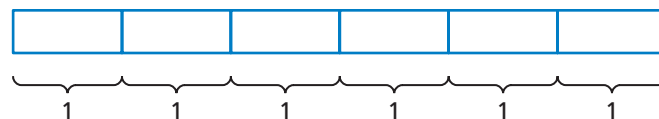
- c. How many three-fourths are in 3?



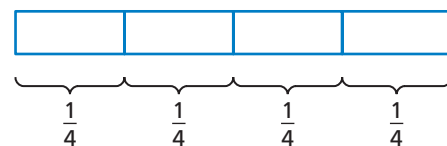
- d. How many four-fifths are in 8?



- e. How many three-tenths are in 6?



- f. How many halves are in a fourth?




What Is Your Answer?

4. **IN YOUR OWN WORDS** How do you divide by a fraction? Give an example.

Practice

Use what you learned about dividing fractions to complete Exercises 11–18 on page 75.

Key Vocabulary 
reciprocals, p. 72

Two numbers whose product is 1 are **reciprocals**. To write a reciprocal of a number, write the number as a fraction. Then invert the fraction.

The Meaning of a Word ● Invert

When you **invert** a glass, you turn it over.



EXAMPLE 1 Writing Reciprocals

Study Tip

When any number is multiplied by 0, the product is 0. So, the number 0 does not have a reciprocal.

	Original Number	Fraction	Reciprocal	Check
a.	$\frac{3}{5}$	$\frac{3}{5}$	$\frac{5}{3}$	$\frac{3}{5} \times \frac{5}{3} = 1$
b.	$\frac{9}{5}$	$\frac{9}{5}$	$\frac{5}{9}$	$\frac{9}{5} \times \frac{5}{9} = 1$
c.	2	$\frac{2}{1}$	$\frac{1}{2}$	$\frac{2}{1} \times \frac{1}{2} = 1$

● On Your Own

Write the reciprocal of the number.

1. $\frac{3}{4}$

2. 5

3. $\frac{7}{2}$

4. $\frac{4}{9}$

Now You're Ready
Exercises 7–10

Key Idea

Dividing Fractions

Words To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Numbers $\frac{1}{5} \div \frac{3}{4} = \frac{1}{5} \times \frac{4}{3}$

Algebra $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$, where $b, c,$ and $d \neq 0$

EXAMPLE 2 Dividing a Fraction by a Fraction

Find $\frac{1}{6} \div \frac{2}{3}$.

$$\frac{1}{6} \div \frac{2}{3} = \frac{1}{6} \times \frac{3}{2}$$

Multiply by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.

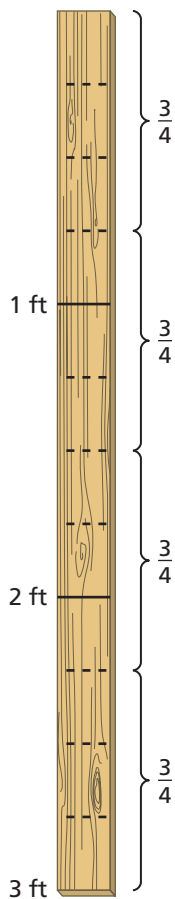
$$= \frac{1 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 2}$$

Multiply fractions. Divide out the common factor 3.

$$= \frac{1}{4}$$

Simplify.

EXAMPLE 3 Dividing a Whole Number by a Fraction



A piece of wood is 3 feet long. How many $\frac{3}{4}$ -foot pieces can be cut from the piece of wood?

Method 1: Draw a diagram. Mark each foot on the diagram. Then divide each foot into $\frac{1}{4}$ -foot sections.

Count the number of $\frac{3}{4}$ -foot pieces of wood. There are four.

∴ So, four $\frac{3}{4}$ -foot pieces can be cut from the piece of wood.

Method 2: Divide 3 by $\frac{3}{4}$ to find the number of $\frac{3}{4}$ -foot pieces.

$$3 \div \frac{3}{4} = 3 \times \frac{4}{3}$$

Multiply by the reciprocal of $\frac{3}{4}$, which is $\frac{4}{3}$.

$$= \frac{\overset{1}{\cancel{3}} \times 4}{\underset{1}{\cancel{3}}}$$

Multiply. Divide out the common factor 3.

$$= 4$$

Simplify.

∴ So, four $\frac{3}{4}$ -foot pieces can be cut from the piece of wood.

On Your Own

Divide. Write the answer in simplest form.

5. $\frac{2}{7} \div \frac{1}{3}$

6. $\frac{1}{2} \div \frac{1}{8}$

7. $\frac{3}{8} \div \frac{1}{4}$

8. $\frac{2}{5} \div \frac{3}{10}$

9. How many $\frac{1}{2}$ -foot pieces can be cut from a 7-foot piece of wood?

Now You're Ready
Exercises 11–26

EXAMPLE 4 Evaluating an Algebraic Expression

Evaluate $a \div b$ when $a = \frac{4}{5}$ and $b = 2$.

$$a \div b = \frac{4}{5} \div 2$$

Substitute $\frac{4}{5}$ for a and 2 for b .

$$= \frac{4}{5} \times \frac{1}{2}$$

Multiply by the reciprocal of 2, which is $\frac{1}{2}$.

$$= \frac{\overset{2}{\cancel{4}} \times 1}{5 \times \cancel{2}_1}$$

Multiply fractions. Divide out the common factor 2.

$$= \frac{2}{5}$$

Simplify.

On Your Own

 **Now You're Ready**
Exercises 32–35

Evaluate the expression $x \div y$ for the given values of x and y .

10. $x = \frac{1}{2}, y = 3$

11. $x = \frac{2}{3}, y = 10$

12. $x = \frac{5}{8}, y = 4$

13. $x = 4, y = \frac{1}{3}$

EXAMPLE 5 Using Order of Operations

Evaluate $\frac{3}{8} + \frac{5}{6} \div 5$.

$$\frac{3}{8} + \frac{5}{6} \div 5 = \frac{3}{8} + \frac{5}{6} \times \frac{1}{5}$$

Multiply by the reciprocal of 5, which is $\frac{1}{5}$.

$$= \frac{3}{8} + \frac{\overset{1}{\cancel{5}} \times 1}{6 \times \cancel{5}_1}$$

Multiply $\frac{5}{6}$ and $\frac{1}{5}$. Divide out the common factor 5.

$$= \frac{3}{8} + \frac{1}{6}$$

Simplify.

$$= \frac{9}{24} + \frac{4}{24}$$

Rewrite fractions using the LCD 24.

$$= \frac{13}{24}$$

Add.

On Your Own

 **Now You're Ready**
Exercises 47–55

Evaluate the expression.

14. $\frac{4}{5} + \frac{2}{5} \div 4$

15. $\frac{3}{8} \div \frac{3}{4} - \frac{1}{6}$

16. $\frac{8}{9} \div 2 \div 8$

2.5 Exercises

Vocabulary and Concept Check

- OPEN-ENDED** Write a fraction and its reciprocal.
- WHICH ONE DOESN'T BELONG?** Which of the following does *not* belong with the other three? Explain your reasoning.

$$\frac{1}{3}$$

$$\frac{1}{6}$$

$$\frac{2}{9}$$

$$\frac{1}{8}$$

MATCHING Match the expression with its value.

3. $\frac{2}{5} \div \frac{8}{15}$

4. $\frac{8}{15} \div \frac{2}{5}$

5. $\frac{2}{15} \div \frac{8}{5}$

6. $\frac{8}{5} \div \frac{2}{15}$

A. $\frac{1}{12}$

B. $\frac{3}{4}$

C. 12

D. $1\frac{1}{3}$

Practice and Problem Solving

Write the reciprocal of the number.

1 7. 8

8. $\frac{6}{7}$

9. $\frac{2}{5}$

10. $\frac{8}{11}$

Divide. Write the answer in simplest form.

2 3 11. $\frac{1}{8} \div \frac{1}{4}$

12. $\frac{5}{6} \div \frac{2}{7}$

13. $12 \div \frac{3}{4}$

14. $8 \div \frac{2}{5}$

15. $\frac{3}{7} \div 6$

16. $\frac{12}{25} \div 4$

17. $\frac{2}{9} \div \frac{2}{3}$

18. $\frac{8}{15} \div \frac{4}{5}$

19. $\frac{1}{3} \div \frac{1}{9}$

20. $\frac{7}{10} \div \frac{3}{8}$

21. $\frac{14}{27} \div 7$

22. $\frac{5}{8} \div 15$


23. $\frac{27}{32} \div \frac{7}{8}$


24. $\frac{4}{15} \div \frac{10}{13}$

25. $9 \div \frac{4}{9}$

26. $10 \div \frac{5}{12}$

ERROR ANALYSIS Describe and correct the error in finding the quotient.

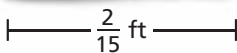
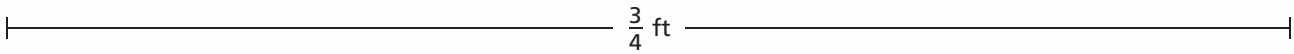
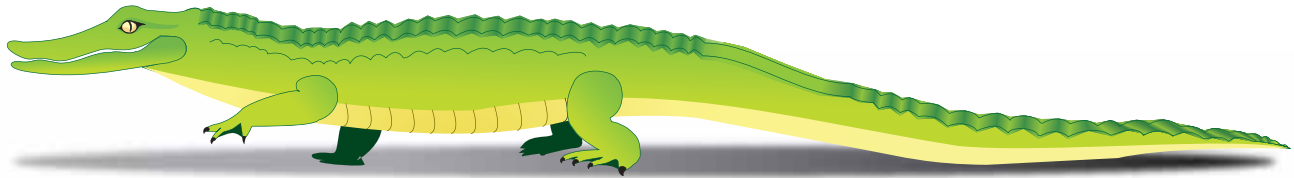
27. 
$$\begin{aligned} \frac{4}{7} \div \frac{13}{28} &= \frac{4}{7} \times \frac{13}{28} \\ &= \frac{\cancel{4}^1 \times 13}{7 \times \cancel{28}_7} \\ &= \frac{13}{49} \end{aligned}$$

28. 
$$\begin{aligned} \frac{2}{5} \div \frac{8}{9} &= \frac{5}{2} \times \frac{8}{9} \\ &= \frac{5 \times \cancel{8}^4}{\cancel{2}_1 \times 9} \\ &= \frac{20}{9} \end{aligned}$$

29. **REASONING** How can you use estimation to show that the quotient in Exercise 28 is incorrect?

30. **APPLE PIE** You have $\frac{3}{5}$ of an apple pie. You divide the remaining pie into five equal slices. What fraction of the original pie is each slice?

31. **ANIMALS** How many times longer is the baby alligator than the baby gecko?



Evaluate the expression when $a = \frac{1}{4}$, $b = \frac{5}{8}$, and $c = 2$.

- 4 32. $a \div b$ 33. $b \div c$ 34. $c \div a$ 35. $b \div a$

Determine whether the numbers are reciprocals. If not, write the reciprocal of each number.

36. $9, \frac{1}{9}$ 37. $\frac{4}{5}, \frac{10}{8}$ 38. $\frac{5}{6}, \frac{15}{18}$ 39. $\frac{6}{5}, \frac{5}{6}$

Copy and complete the statement.

40. $\frac{5}{12} \times \square = 1$ 41. $3 \times \square = 1$ 42. $7 \div \square = 56$

Without finding the quotient, copy and complete the statement using $<$, $>$, or $=$. Explain your reasoning.

43. $5 \div \frac{7}{9} \square 5$ 44. $\frac{3}{7} \div 1 \square \frac{3}{7}$
 45. $8 \div \frac{3}{4} \square 8$ 46. $\frac{5}{6} \div \frac{7}{8} \square \frac{5}{6}$

Evaluate the expression.

- 5 47. $\frac{1}{6} \div 6 \div 6$ 48. $\frac{7}{12} \div 14 \div 6$ 49. $\frac{3}{5} \div \frac{4}{7} \div \frac{9}{10}$
 50. $4 \div \frac{8}{9} - \frac{1}{2}$ 51. $\frac{3}{4} + \frac{5}{6} \div \frac{2}{3}$ 52. $\frac{7}{8} - \frac{3}{8} \div 9$
 53. $\frac{9}{16} \div \frac{3}{4} \cdot \frac{2}{13}$ 54. $\frac{3}{14} \cdot \frac{2}{5} \div \frac{6}{7}$ 55. $\frac{10}{27} \cdot \left(\frac{3}{8} \div \frac{5}{24} \right)$

56. **REASONING** Use a model to evaluate the quotient $\frac{1}{2} \div \frac{1}{6}$. Explain.

57. **NUMBER SENSE** When is the reciprocal of a fraction a whole number? Explain.

58. **BUDGETS** The table shows the portions of a family budget that are spent on several expenses.

Expense	Portion of Budget
Housing	$\frac{1}{4}$
Food	$\frac{1}{12}$
Automobiles	$\frac{1}{15}$
Recreation	$\frac{1}{40}$

- How many times more is the expense for housing than for automobiles?
- How many times more is the expense for food than for recreation?
- The expense for automobile fuel is $\frac{1}{60}$ of the total expenses. What fraction of the automobile expense is spent on fuel?



59. **GLAZING** You have 6 pints of glaze. It takes $\frac{7}{8}$ pint to glaze a bowl and $\frac{9}{16}$ pint to glaze a plate.

- How many bowls could you glaze? How many plates could you glaze?
- You want to glaze 5 bowls and then use the rest for plates. How many plates can you glaze? How much glaze will be left over?
- How many of each object could you glaze so that there is no glaze left over? Explain how you found your answer.

60. **Reasoning** A water tank is $\frac{1}{8}$ full. The tank is $\frac{3}{4}$ full when 42 gallons of water are added to the tank.

- How much water can the tank hold?
- How much water was originally in the tank?
- How much water is in the tank when it is $\frac{1}{2}$ full?



Fair Game Review What you learned in previous grades & lessons

Estimate the quotient. (Section 2.1)

61. $12\frac{1}{9} \div 3\frac{4}{5}$

62. $71\frac{2}{3} \div 8\frac{1}{4}$

63. $90\frac{2}{7} \div 9\frac{3}{8}$

64. $47\frac{3}{4} \div 7\frac{5}{6}$

65. **MULTIPLE CHOICE** The expression $3m$ represents the cost of renting m movies. What is the cost of renting four movies? (Section 1.1)

(A) \$7

(B) \$9

(C) \$12

(D) \$27