

2.8 Writing Fractions as Decimals

Essential Question How can you tell from the denominator of a fraction if its decimal form is terminating or repeating?

1 EXAMPLE: Writing a Fraction as a Decimal

Write the fraction as a decimal. Is it terminating or repeating?

a. $\frac{3}{8}$

1. To divide 3 by 8, place a decimal point after 3.

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.000} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

2. Place as many zeros as necessary to complete the division.

3. Place the decimal point directly above the decimal point you placed in Step 1.

☘ The division terminates. So, $\frac{3}{8} = 0.375$ is a terminating decimal.

b. $\frac{2}{3}$

1. To divide 2 by 3, place a decimal point after 2.

$$\begin{array}{r} 0.666 \\ 3 \overline{)2.000} \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

2. Place as many zeros as necessary to see a pattern in the remainder.

3. Place the decimal point directly above the decimal point you placed in Step 1.

☘ The division does not end. So, $\frac{2}{3} = 0.666\dots$ is a **repeating decimal**.

Inductive Reasoning

Write the fraction as a decimal. Is it terminating or repeating?

2. $\frac{3}{4}$

3. $\frac{1}{16}$

4. $\frac{1}{6}$

5. $\frac{2}{9}$

6. $\frac{7}{20}$

7. $\frac{7}{8}$

8. $\frac{5}{12}$

9. $\frac{21}{40}$

Use estimation to match the fraction with its decimal. Then use a calculator to check your answer.

10. $\frac{5}{6}$

11. $\frac{1}{3}$

12. $\frac{5}{8}$

13. $\frac{3}{16}$

A. 0.625

B. 0.1875

C. 0.333...

















D. 0.83333...

14. In $\frac{1}{7} = 0.1428571428571428571 \dots$, what are the repeating digits?

Can you find another fraction that has at least six digits that repeat?





























































What Is Your Answer?

15. a. Describe the denominators of fractions that can be written as terminating decimals.
 b. Describe the denominators of fractions that can be written as repeating decimals.
16. **IN YOUR OWN WORDS** How can you tell from the denominator of a fraction if its decimal form is terminating or repeating?
17. **Reasoning** The Mayan number system was base 20. In a base 20 system, describe the denominators of fractions that would be represented by terminating decimals and by repeating decimals.

0 	1 •	2 ••	3 •••	4 ••••
5 	6 	7 	8 	9 
10 	11 	12 	13 	14 
15 	16 	17 	18 	19 



18. **Reasoning** The Babylonian number system was base 60. In a base 60 system, describe the denominators of fractions that would be represented by terminating decimals and by repeating decimals.

1 	11 	21 	31 	41 	51 
2 	12 	22 	32 	42 	52 
3 	13 	23 	33 	43 	53 
4 	14 	24 	34 	44 	54 
5 	15 	25 	35 	45 	55 
6 	16 	26 	36 	46 	56 
7 	17 	27 	37 	47 	57 
8 	18 	28 	38 	48 	58 
9 	19 	29 	39 	49 	59 
10 	20 	30 	40 	50 	60 



Practice

Use what you learned about writing fractions as decimals to complete Exercises 11–18 on page 94.

Key Vocabulary

repeating decimal,
p. 92

When writing a fraction as a decimal, your result is a terminating or repeating decimal. A **repeating decimal** repeats a pattern of one or more digits.

Terminating decimal: $\frac{1}{2} = 0.5$

Repeating decimal: $\frac{2}{3} = 0.666\dots = 0.\overline{6}$

Show that a decimal repeats by using three dots or placing a bar over the digit that repeats.

Key Idea

Method 1: Writing Fractions as Decimals

To write a fraction as a decimal, divide the numerator by the denominator.

EXAMPLE 1 Writing Fractions as Decimals

a. Write $\frac{1}{8}$ as a decimal.

b. Write $\frac{5}{6}$ as a decimal.

$$\begin{array}{r} 0.125 \\ 8 \overline{)1.000} \\ \underline{-8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Place the decimal point.

Place zeros to complete the division.

Remainder is 0. The decimal terminates.

$$\therefore \frac{1}{8} = 1 \div 8 = 0.125$$

$$\begin{array}{r} 0.833 \\ 6 \overline{)5.000} \\ \underline{-48} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array}$$

Place the decimal point.

Place zeros to see a pattern in the remainder.

The remainder 2 keeps repeating.

$$\therefore \frac{5}{6} = 5 \div 6 = 0.833\dots \text{ or } 0.8\overline{3}$$

Reading

One way to say the repeating decimal $0.8\overline{3}$ is "zero point eight three, with the three repeating."

On Your Own

Write the fraction as a decimal. Is it terminating or repeating?

1. $\frac{3}{5}$

2. $\frac{1}{9}$

3. $\frac{9}{20}$

4. $\frac{8}{5}$

Key Idea

Method 2: Writing Fractions as Decimals

To write a fraction as a decimal, write an equivalent fraction (if possible) whose denominator is 10, 100, or 1000.

EXAMPLE 2 Writing a Fraction as a Decimal

Write $\frac{3}{20}$ as a decimal.

$$\begin{aligned} \frac{3}{20} &= \frac{3 \times 5}{20 \times 5} && \leftarrow \text{Because } 20 \times 5 = 100, \text{ multiply the numerator and denominator by } 5. \\ &= \frac{15}{100} && \text{Multiply.} \\ &= 0.15 && \frac{15}{100} \text{ is read as "fifteen hundredths."} \end{aligned}$$

20 is a factor of 100.

EXAMPLE 3 Real-Life Application

You run the 40-yard dash in $6\frac{21}{25}$ seconds. Your teammate runs it in the time shown. Who is faster and by how much?

Write $6\frac{21}{25}$ as a decimal and compare it to 6.9.



$$\begin{aligned} 6\frac{21}{25} &= 6 + \frac{21}{25} && \text{Write the mixed number as a sum.} \\ &= 6 + \frac{21 \times 4}{25 \times 4} && \leftarrow \text{Because } 25 \times 4 = 100, \text{ multiply the numerator and denominator by } 4. \\ &= 6 + \frac{84}{100} && \text{Multiply.} \\ &= 6 + 0.84 = 6.84 && \frac{84}{100} \text{ is read as "eighty-four hundredths."} \end{aligned}$$

$6.84 < 6.9$. The difference is $6.9 - 6.84 = 0.06$ second.

∴ So, you are faster by 0.06 second.

On Your Own

Write the fraction as a decimal.

5. $\frac{4}{5}$ 6. $\frac{11}{20}$ 7. $\frac{7}{50}$ 8. $\frac{47}{250}$

9. In Example 3, Pedro runs the 40-yard dash in $6\frac{37}{50}$ seconds. Is he the fastest? Explain.

Now You're Ready
Exercises 11–26

Vocabulary and Concept Check

- NUMBER SENSE** When using division to write a fraction as a decimal, when do you stop dividing?
- WHICH ONE DOESN'T BELONG?** Which fraction does *not* belong with the other three? Explain your reasoning.

$$\frac{1}{4}$$

$$\frac{1}{5}$$

$$\frac{1}{6}$$

$$\frac{1}{8}$$

Tell whether the decimal is repeating or terminating.

3. 0.625

4. $0.1\bar{3}$

5. $7.\bar{4}$

6. 0.470

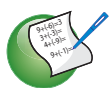
Rewrite the repeating decimal using bar notation.

7. 0.1111...

8. 3.4444...

9. 0.5333...

10. 0.1666...



Practice and Problem Solving

Write the fraction as a decimal.

1 2 11. $\frac{7}{10}$

12. $\frac{5}{6}$

13. $\frac{31}{50}$

14. $\frac{2}{15}$

15. $\frac{17}{18}$

16. $\frac{23}{40}$

17. $\frac{21}{25}$

18. $\frac{11}{45}$

19. $\frac{3}{20}$

20. $\frac{7}{18}$

21. $\frac{3}{8}$

22. $\frac{6}{25}$

23. $\frac{19}{30}$

24. $\frac{51}{40}$

25. $\frac{114}{25}$

26. $\frac{22}{15}$

27. **ERROR ANALYSIS** Describe and correct the error in writing $\frac{8}{15}$ as a decimal.

28. **GOLDFISH** The length of a goldfish is $\frac{5}{16}$ foot. Write the length of the goldfish as a decimal.

29. **BASEBALL** Your batting average for a baseball season is $\frac{11}{15}$. Write your batting average as a decimal.

X

$$\begin{array}{r}
 0.533 \dots \\
 15 \overline{) 8.000} \\
 \underline{75} \\
 50 \\
 \underline{45} \\
 50 \\
 \underline{45} \\
 5
 \end{array}$$

$\frac{8}{15} = 0.\overline{53}$

Copy and complete the statement using $<$, $>$, or $=$.

30. $\frac{7}{8}$ 0.85

31. $\frac{5}{12}$ 0.4

32. 1.95 $\frac{39}{20}$

Write the number as a fraction. Then write the fraction as a decimal.

33. five-eighths
 34. eleven-twelfths
 35. nine-fourteenths
 36. sixty-three twentieths
37. **OPEN-ENDED** Find a fraction whose decimal value is between 0.5 and 0.65. Write the fraction and its decimal equivalent.
38. **ENDANGERED SPECIES** The table shows shell lengths of four sea turtles that are on the endangered species list.

Sea Turtle	Shell Length
Green	$2\frac{9}{20}$ ft
Hawksbill	$2\frac{1}{9}$ ft
Kemp's Ridley	$2\frac{1}{10}$ ft
Leatherback	$2\frac{5}{11}$ ft



- a. Convert the shell lengths to decimals. Then order the shell lengths from least to greatest.
- b. How much longer is the leatherback sea turtle than the green sea turtle?
39. **PATTERNS** Use the following pattern.

$$\frac{0}{9} = 0.000000\dots, \quad \frac{1}{9} = 0.111111\dots, \quad \frac{2}{9} = 0.222222\dots$$

- a. Complete the pattern for $\frac{3}{9}$ through $\frac{9}{9}$.
- b. Two of the decimals in the pattern have terminating decimal forms. One is $0.000000\dots$, which is simply 0. The other is $0.999999\dots$. What is the terminating form of this decimal? Explain your reasoning.



Write the fraction as a decimal.

40. $\frac{115}{333}$ 41. $\frac{115}{33,300}$ 42. $\frac{1001}{8000}$ 43. $\frac{1001}{800,000}$



Fair Game Review

what you learned in previous grades & lessons

Use rounding to estimate the sum or difference. (*Skills Review Handbook*)

44. $4.8 + 6.4$ 45. $10.7 - 3.8$ 46. $2.16 + 7.44$ 47. $16.58 - 5.26$
48. **MULTIPLE CHOICE** Which expression is equivalent to $9(3 + x + 5)$? (*Section 1.4*)

- (A) $x + 32$ (B) $9x + 32$ (C) $9x + 72$ (D) $27x + 45$