8.1 Writing and Graphing Inequalities

Essential Question How can you use a number line to represent solutions of an inequality?

1 ACTIVITY: Understanding Inequality Statements

Work with a partner.

a. **Consider the statement "Your friend is more than 3 minutes late."**

   Circle each number that makes the statement true.

   $$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

   • Write four other numbers that make the statement true.

b. **Consider the statement "The temperature is at most 2 degrees."**

   • Can the temperature be exactly 2 degrees? Explain.

   • Circle each number that makes the statement true.

   $$-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

   • Write four other numbers that make the statement true.

c. **Consider the statement "You need at least 4 pieces of paper for your math homework."**

   • Can you have exactly 4 pieces of paper? Explain.

   • Circle each number that makes the statement true.

   $$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$$

   • Write four other numbers that make the statement true.

d. **Consider the statement "After playing a video game for 20 minutes, you have fewer than 6 points."**

   • Circle each number that makes the statement true.

   $$-2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7$$

   • Write four other numbers that make the statement true.

2 ACTIVITY: Understanding Inequality Symbols

Work with a partner.

a. Consider the statement "x is a number such that $x < 2$."

  - Can the number be exactly 2? Explain.

  - Circle each number that makes the statement true.

    $$-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

  - Write four other numbers that make the statement true.

b. Consider the statement "x is a number such that $x \geq 1$."

  - Can the number be exactly 1? Explain.
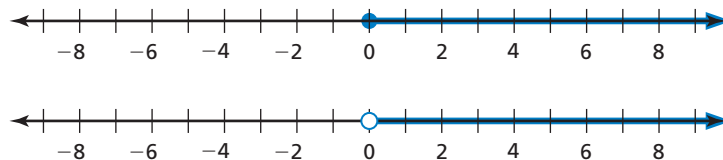
  - Circle each number that makes the statement true.

    $$-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$$

  - Write four other numbers that make the statement true.

3 ACTIVITY: How Close Can You Come to 0?

Work with a partner.

a. Which number line shows $x > 0$? Which number line shows $x \geq 0$? Explain your reasoning.

b. Write the least positive number you can think of that is still a solution of the inequality $x > 0$. Explain your reasoning.

What Is Your Answer?

4. IN YOUR OWN WORDS How can you use a number line to represent solutions of an inequality?

5. Write an inequality. Graph all solutions of your inequality on a number line.

Practice

Use what you learned about graphing inequalities to complete Exercises 17–20 on page 333.

Key Vocabulary 🔊
inequality, *p. 330*
solution of an
   inequality, *p. 331*
solution set, *p. 331*
graph of an
   inequality, *p. 332*

An **inequality** is a mathematical sentence that compares expressions. It contains the symbols $<$, $>$, $\leq$, or $\geq$. To write an inequality, look for the following phrases to determine where to place the inequality symbol.

Inequality Symbols				
Symbol	$<$	$>$	$\leq$	$\geq$
Key Phrases	• is less than • is fewer than	• is greater than • is more than	• is less than or equal to • is at most • is no more than	• is greater than or equal to • is at least • is no less than

EXAMPLE 1 — Writing Inequalities

Write the word sentence as an inequality.

a. A number c is less than 4.

A number c is less than 4.

c $<$ 4

⋮• An inequality is $c < 4$.

b. A number k plus 5 is greater than or equal to 8.

A number k plus 5 is greater than or equal to 8.

$k + 5$ $\geq$ 8

⋮• An inequality is $k + 5 \geq 8$.

c. Four times a number q is at most 16.

Four times a number q is at most 16.

$4q$ $\leq$ 16

⋮• An inequality is $4q \leq 16$.

🔴 On Your Own

Now You're Ready
Exercises 5–10

Write the word sentence as an inequality.

1. A number n is greater than 1.

2. Twice a number p is fewer than 7.

3. A number w minus 3 is less than or equal to 10.

4. A number z divided by 2 is at least 6.

🔊 **Multi-Language Glossary at** BigIdeasMath✓com.

A **solution of an inequality** is a value that makes the inequality true. An inequality can have more than one solution. The set of all solutions of an inequality is called the **solution set**.

Value of x	$x + 3 \leq 7$	Is the inequality true?
3	$3 + 3 \overset{?}{\leq} 7$ $6 \leq 7$ ✓	yes
4	$4 + 3 \overset{?}{\leq} 7$ $7 \leq 7$ ✓	yes
5	$5 + 3 \overset{?}{\leq} 7$ $8 \nleq 7$ ✗	no

Reading

The symbol $\nleq$ means "is not less than or equal to."

EXAMPLE 2 Checking Solutions

Tell whether the given value is a solution of the inequality.

a. $x + 1 > 7$; $x = 8$

$\quad\quad x + 1 > 7$ — Write the inequality.

$\quad\quad 8 + 1 \overset{?}{>} 7$ — Substitute 8 for x.

$\quad\quad\quad\quad 9 > 7$ ✓ — Add. 9 is greater than 7.

⋮ So, 8 is a solution of the inequality.

b. $7y < 27$; $y = 4$

$\quad\quad 7y < 27$ — Write the inequality.

$\quad\quad 7(4) \overset{?}{<} 27$ — Substitute 4 for y.

$\quad\quad 28 \nless 27$ ✗ — Multiply. 28 is *not* less than 27.

⋮ So, 4 is *not* a solution of the inequality.

c. $\dfrac{z}{3} \geq 5$; $z = 15$

$\quad\quad \dfrac{z}{3} \geq 5$ — Write the inequality.

$\quad\quad \dfrac{15}{3} \overset{?}{\geq} 5$ — Substitute 15 for z.

$\quad\quad 5 \geq 5$ ✓ — Divide. 5 is greater than or equal to 5.

⋮ So, 15 is a solution of the inequality.

On Your Own

Now You're Ready
Exercises 11–16

Tell whether 3 is a solution of the inequality.
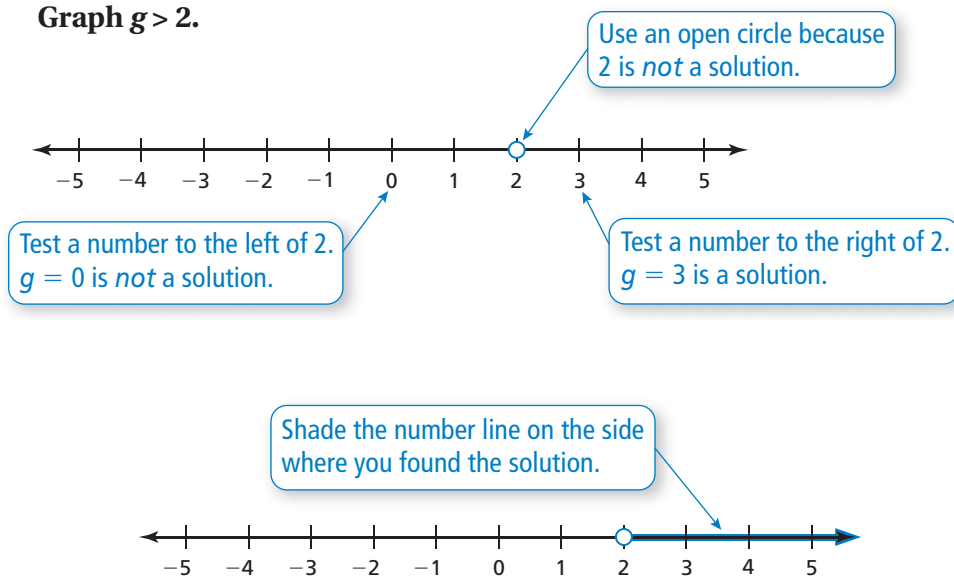
5. $b + 4 < 6$ **6.** $9 - n \geq 6$ **7.** $18 \div x \leq 10$

The **graph of an inequality** shows all of the solutions of the inequality on a number line. An open circle ○ is used when a number is *not* a solution. A closed circle ● is used when a number is a solution. An arrow to the left or right shows that the graph continues in that direction.

EXAMPLE 3 **Graphing an Inequality**

Graph $g > 2$.

> Use an open circle because 2 is *not* a solution.

> Test a number to the left of 2.
> $g = 0$ is *not* a solution.

> Test a number to the right of 2.
> $g = 3$ is a solution.

> Shade the number line on the side where you found the solution.

EXAMPLE 4 **Real-Life Application**

The NASA Solar Probe can withstand temperatures up to and including 2600°F. Write and graph an inequality that represents the temperatures the probe can withstand.

Words	temperatures	up to and including	2600°F

Variable   Let t be the temperatures the probe can withstand.

Inequality	t	$\leq$	2600

∴ An inequality is $t \leq 2600$.

On Your Own

Now You're Ready
Exercises 25–36

Graph the inequality on a number line.

8. $a < 4$ **9.** $f \leq 7$ **10.** $n > 0$ **11.** $p \geq -3$

Write and graph an inequality for the situation.

12. A cruise ship can carry at most 3500 passengers.

13. A board game is designed for ages 12 and up.

✓ Vocabulary and Concept Check

1. **VOCABULARY** How are "greater than" and "greater than or equal to" similar? How are they different?

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Write "both" inequalities.

    A number n is at most 3.

    A number n is at least 3.

    A number n is less than or equal to 3.

    A number n is no more than 3.

3. **WRITING** Explain how the graph of $x \leq 6$ is different from the graph of $x < 6$.

4. **WRITING** Are the graphs of $x \leq 5$ and $5 \geq x$ the same or different? Explain.

✏️ Practice and Problem Solving

Write the word sentence as an inequality.

① 5. A number k is less than 10.

6. A number a is more than 6.

7. A number z is fewer than $\dfrac{3}{4}$.

8. A number b is at least -3.

9. One plus a number y is no more than 13.

10. A number x divided by 3 is at most 5.

Tell whether the given value is a solution of the inequality.

② 11. $x - 1 \leq 7$; $x = 6$

12. $y + 5 < 13$; $y = 17$

13. $3z > 6$; $z = 3$

14. $\dfrac{b}{2} \geq 6$; $b = 10$
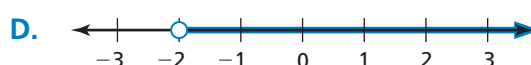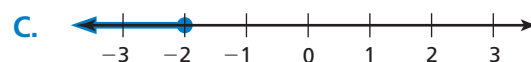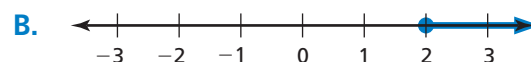
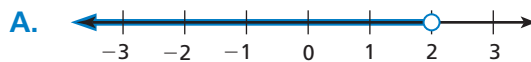15. $c + 2.5 < 4.3$; $c = 1.8$

16. $a \leq 0$; $a = -5$
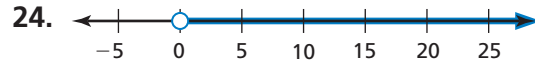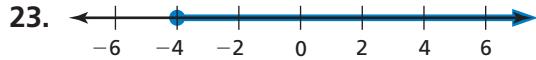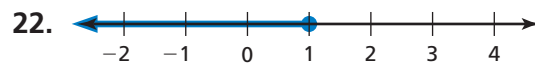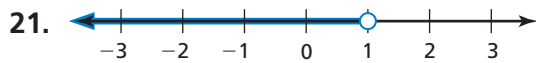
Match each inequality with its graph.

17. $x \geq 2$ 18. $x < 2$ 19. $x > -2$ 20. $x \leq -2$

A.
```
←——+——+——+——+——+——○——+——→
  −3  −2  −1   0   1   2   3
```

B.
```
←——+——+——+——+——+——●————————→
  −3  −2  −1   0   1   2   3
```

C.
```
←————————●——+——+——+——+——+——→
  −3  −2  −1   0   1   2   3
```

D.
```
←——+——○——+——+——+——+——+——→
  −3  −2  −1   0   1   2   3
```

Write an inequality and a word sentence that represent the graph.

21.

22.

23.

24.

Graph the inequality on a number line.

③ 25. $a > 4$

26. $n \geq 8$

27. $3 \geq x$

28. $y < \dfrac{1}{2}$

29. $x < \dfrac{2}{9}$
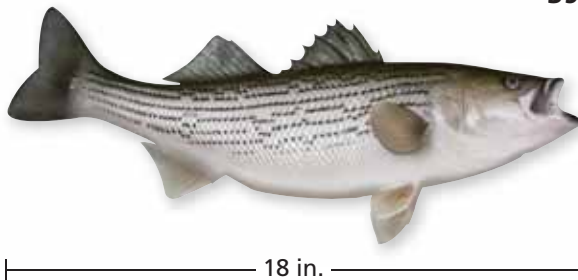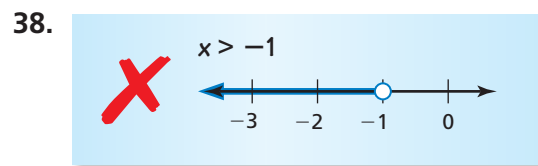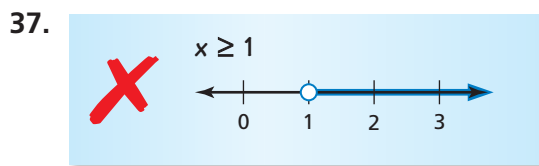
30. $-3 \geq c$

31. $m > -5$

32. $b \geq 0$

33. $f < 1.5$

34. $t \geq -\dfrac{1}{2}$

35. $p > -1.6$

36. $z \leq -\dfrac{7}{3}$

ERROR ANALYSIS **Describe and correct the error in graphing the inequality.**

37.

$x \geq 1$

38.

$x > -1$

39. **FISHING** You are fishing and are allowed to catch at most 3 striped bass. Each striped bass must be no less than 18 inches in length.

18 in.

 a. Write and graph an inequality to represent the number of striped bass you are allowed to catch.

 b. Write and graph an inequality to represent the length of each striped bass you are allowed to catch.

40. **LOW SODIUM** For a food to be labeled *low sodium,* there must be no more than 140 milligrams of sodium per serving.

 a. Write and graph an inequality to represent the amount of sodium in a low sodium serving.

 b. Write and graph an inequality to represent the amount of sodium in a serving that does not qualify as low sodium.

 c. Does the food represented by the label qualify as a low sodium food? Explain.

Nutrition Facts

Serving Size ½ cup (114g)
Servings Per Container 4

Amount Per Serving

Calories 90   Calories from fat 30

% **Daily Value***

Total Fat 3g — 5%
    Saturated Fat 0g — 0%
Cholesterol 0mg — 0%
Sodium 300mg — 13%
Total Carbohydrate 13g — 4%
    Dietary Fiber 3g — 12%
    Sugars 3g
Protein 3g

Vitamin A 80% • Vitamin C 60%
Calcium 4% • Iron 4%

41. SHOPPING You have $33. You want to buy a necklace and one other item from the list.

    a. Write an inequality to represent the situation.

    b. Can the other item be a T-shirt? Explain.

    c. Can the other item be a book? Explain.

Item	Price (with tax)
T-shirt	$ 15
Book	$ 20
DVD	$ 13
Necklace	$ 16
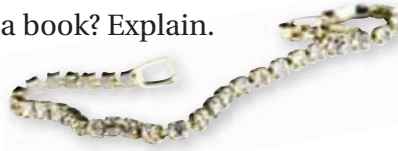
Determine whether the statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

42. A number that is a solution of the inequality $x > 5$ is also a solution of the inequality $x \geq 5$.

43. A number that is a solution of the inequality $5 \leq x$ is also a solution of the inequality $x > 5$.

44. BUS RIDE A bus ride costs $1.50. A 30-day bus pass costs $36. Write an inequality to represent the number of bus rides you would need to take for the bus pass to be a better deal.

45. MOVIE THEATER Fifty people are seated in a movie theater. The maximum capacity of the theater is 425 people. Write an inequality to represent the number of additional people who can still be seated.

46. ✸**Critical Thinking**✸ The map shows the elevations above sea level for an area of land.
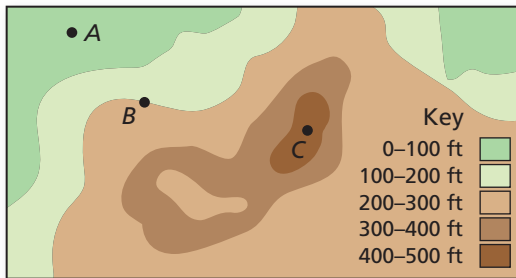
    a. Graph the possible elevations of *A*. Write the set of elevations as two inequalities.

    b. Graph the possible elevations of *C*. How can you write this set of elevations as a single inequality? Explain.

    c. What is the elevation of *B*? Explain.

Key
0–100 ft
100–200 ft
200–300 ft
300–400 ft
400–500 ft

🖍️ **Fair Game Review** *What you learned in previous grades & lessons*

Solve the equation. Check your solution.

47. $x + 3 = 12$ **48.** $x - 6 = 8$ **49.** $16 + x = 44$ **50.** $7.6 = x - 6.5$

51. MULTIPLE CHOICE A stack of boards is 24 inches high. Each board is $\frac{3}{8}$ of an inch thick. How many boards are in the stack?

    Ⓐ $\frac{1}{6}$ Ⓑ $\frac{1}{9}$ Ⓒ 9   Ⓓ 64