Multiplying and Dividing Rational Numbers

Essential Question How can you use operations with rational numbers in a story?

1 EXAMPLE: Writing a Story

Write a story that uses addition, subtraction multiplication, or division of rational numbers. Draw pictures for your story.

There are many possible stories. Here is an example.

24 Lemons	–$11.75
5 cups sugar	–$1.50
30 plastic glasses	–$1.50
18 sales ($0.50 each)	$9.00
PROFIT	–$5.75

LEMONADE

All You Can Drink For 50¢!

Lauryn decides to earn some extra money. She sets up a lemonade stand. To get customers, she uses big plastic glasses and makes a sign saying "All you can drink for 50¢!"

Lauryn can see that her daily profit is negative. But, she decides to keep trying. After one week, she has the same profit each day.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
–$5.75	–$5.75	–$5.75	–$5.75	–$5.75	–$5.75	–$5.75

Lauryn is frustrated. Her profit for the first week is

$$7(-5.75) = (-5.75) + (-5.75) + (-5.75) + (-5.75) + (-5.75) + (-5.75) + (-5.75)$$
$$= -40.25.$$

She realizes that she has too many customers who are drinking a second and even a third glass of lemonade. So, she decides to try a new strategy. Soon, she has a customer. He buys a glass of lemonade and drinks it.

He hands the empty glass to Lauryn and says "*That was great. I'll have another glass.*" Today, Lauryn says "*That will be 50¢ more, please.*" The man says "*But, you only gave me one glass and the sign says 'All you can drink for 50¢!'*" Lauryn replies, "*One glass IS all you can drink for 50¢.*"

With her new sales strategy, Lauryn starts making a profit of $8.25 per day. Her profit for the second week is

$$7(8.25) = (8.25) + (8.25) + (8.25) + (8.25) + (8.25) + (8.25) + (8.25) = 57.75.$$

Her profit for the two weeks is $-40.25 + 57.75 = \$17.50$. So, Lauryn has made some money. She decides that she is on the right track.

Work with a partner. Write a story that uses addition, subtraction, multiplication, or division of rational numbers.

- At least one of the numbers in the story has to be negative and *not* an integer.
- Draw pictures to help illustrate what is happening in the story.
- Include the solution of the problem in the story.

If you are having trouble thinking of a story, here are some common uses of negative numbers.

- A profit of $-\$15$ is a loss of $\$15$.
- An elevation of -100 feet is a depth of 100 feet below sea level.
- A gain of -5 yards in football is a loss of 5 yards.
- A score of -4 in golf is 4 strokes under par.
- A balance of $-\$25$ in your checking account means the account is overdrawn by $25.

What Is Your Answer?

3. IN YOUR OWN WORDS How can you use operations with rational numbers in a story? You already used rational numbers in your story. Describe another use of a negative rational number in a story.

PUZZLE Read the cartoon. Fill in the blanks using 4s or 8s to make the equation true.
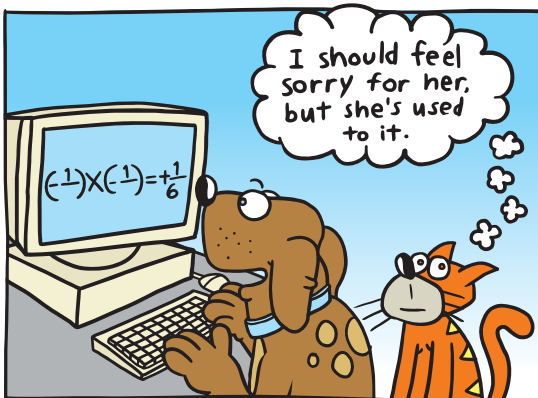
"Dear Mom, I'm in a hurry. To save time I won't be typing any 4's or 8's."

4. $\left(-\dfrac{1}{\boxed{}}\right) + \left(-\dfrac{1}{\boxed{}}\right) = -\dfrac{1}{\boxed{}}$

5. $\left(-\dfrac{1}{\boxed{}}\right) \times \left(-\dfrac{1}{\boxed{}}\right) = \dfrac{1}{6\boxed{}}$

6. $1.\boxed{} \times \left(-0.\boxed{}\right) = -1.\boxed{}\,\boxed{}$

7. $\left(-\dfrac{3}{\boxed{}}\right) \div \left(\dfrac{3}{\boxed{}}\right) = -\dfrac{1}{2}$

8. $-4.\boxed{} \div 2 = -2.\boxed{}$

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🔑 **Key Idea**

Remember

The *reciprocal* of $\frac{a}{b}$ is $\frac{b}{a}$.

Multiplying and Dividing Rational Numbers

Words To multiply or divide rational numbers, use the same rules for signs as you used for integers.

Numbers

$$-\frac{2}{7} \cdot \frac{1}{3} = \frac{-2 \cdot 1}{7 \cdot 3} = \frac{-2}{21} = -\frac{2}{21}$$

$$-\frac{1}{2} \div \frac{4}{9} = \frac{-1}{2} \cdot \frac{9}{4} = \frac{-1 \cdot 9}{2 \cdot 4} = \frac{-9}{8} = -\frac{9}{8}$$

EXAMPLE 1 Dividing Rational Numbers

Find $-5\frac{1}{5} \div 2\frac{1}{3}$.

Estimate $-5 \div 2 = -2\frac{1}{2}$

$$-5\frac{1}{5} \div 2\frac{1}{3} = -\frac{26}{5} \div \frac{7}{3}$$

Write mixed numbers as improper fractions.

$$= \frac{-26}{5} \cdot \frac{3}{7}$$

Multiply by the reciprocal of $\frac{7}{3}$.

$$= \frac{-26 \cdot 3}{5 \cdot 7}$$

Multiply the numerators and the denominators.

$$= \frac{-78}{35}, \text{ or } -2\frac{8}{35}$$

Simplify.

⁘ The quotient is $-2\frac{8}{35}$.

Reasonable? $-2\frac{8}{35} \approx -2\frac{1}{2}$ ✔

EXAMPLE 2 Multiplying Rational Numbers

Find $-2.5 \cdot 3.6$.

$$
\begin{array}{r}
-2.5 \\
\times\ 3.6 \\
\hline
1\ 5\ 0 \\
7\ 5\ 0 \\
\hline
-9.0\ 0
\end{array}
$$

← The decimals have different signs.

← The product is negative.

⁘ The product is -9.

EXAMPLE **3**   Standardized Test Practice

Which number, when multiplied by $-\frac{5}{3}$, gives a product between 5 and 6?

Ⓐ -6 Ⓑ $-3\frac{1}{4}$ Ⓒ $-\frac{1}{4}$ Ⓓ 3

Use the guess, check, and revise method.

Guess 1: Because the product is positive and the known factor is negative, choose a number that is negative. Try Choice Ⓒ.

$$-\frac{1}{4}\left(-\frac{5}{3}\right) = \frac{-1 \cdot (-5)}{4 \cdot 3} = \frac{5}{12}$$

Guess 2: The result of Choice Ⓒ is not between 5 and 6. So, choose another number that is negative. Try Choice Ⓑ.

$$-3\frac{1}{4}\left(-\frac{5}{3}\right) = -\frac{13}{4}\left(-\frac{5}{3}\right) = \frac{-13 \cdot (-5)}{4 \cdot 3} = \frac{65}{12} = 5\frac{5}{12}$$

∴ $5\frac{5}{12}$ is between 5 and 6. So, the correct answer is Ⓑ.

On Your Own

Multiply or divide.

1. $-\frac{6}{5} \div \left(-\frac{1}{2}\right)$ **2.** $\frac{1}{3} \div \left(-2\frac{2}{3}\right)$ **3.** $\left(-\frac{1}{2}\right)^3$

4. $1.8(-5.1)$ **5.** $-6.3(-0.6)$ **6.** $(-1.3)^2$

EXAMPLE 4   Real-Life Application

Account Positions | ⟳			
Stock	**Original Value**	**Current Value**	**Change**
A	600.54	420.15	−180.39
B	391.10	518.38	127.28
C	380.22	99.70	−280.52

An investor owns stocks A, B, and C. What is the mean change in value of the stocks?

$$\text{mean} = \frac{-180.39 + 127.28 + (-280.52)}{3} = \frac{-333.63}{3} = -111.21$$

∴ The mean change in value of the stocks is −$111.21.

On Your Own

7. In Example 4, the change in value of stock D is $568.23. What is the mean change in value of the four stocks?

✓ Vocabulary and Concept Check

1. **WRITING** How is multiplying and dividing rational numbers similar to multiplying and dividing integers?

Find the reciprocal.

2. $-\dfrac{2}{5}$

3. -3

4. $\dfrac{16}{9}$

5. $-2\dfrac{1}{3}$

Tell whether the expression is *positive* or *negative* without evaluating.

6. $-\dfrac{3}{10} \times \left(-\dfrac{8}{15}\right)$

7. $1\dfrac{1}{2} \div \left(-\dfrac{1}{4}\right)$

8. -6.2×8.18

9. $\dfrac{-8.16}{-2.72}$

✎ Practice and Problem Solving

Divide. Write fractions in simplest form.

① 10. $-\dfrac{7}{10} \div \dfrac{2}{5}$

11. $\dfrac{1}{4} \div \left(-\dfrac{3}{8}\right)$

12. $-\dfrac{8}{9} \div \left(-\dfrac{8}{9}\right)$

13. $-\dfrac{1}{5} \div 20$

14. $-2\dfrac{4}{5} \div (-7)$

15. $-10\dfrac{2}{7} \div \left(-4\dfrac{4}{11}\right)$

16. $-9 \div 7.2$

17. $8 \div 2.2$

18. $-3.45 \div (-15)$

19. $-0.18 \div 0.03$

20. $8.722 \div (-3.56)$

21. $12.42 \div (-4.8)$

Multiply. Write fractions in simplest form.

② ③ 22. $-\dfrac{2}{3} \times \dfrac{2}{9}$

23. $-\dfrac{1}{4} \times \left(-\dfrac{4}{3}\right)$

24. $\dfrac{5}{6}\left(-\dfrac{8}{15}\right)$

25. $-2\left(-1\dfrac{1}{4}\right)$

26. $-3\dfrac{1}{3} \cdot \left(-2\dfrac{7}{10}\right)$

27. $\left(-1\dfrac{2}{3}\right)^3$

28. $0.4 \times (-0.03)$

29. $-0.05 \times (-0.5)$

30. $-8(0.09)$

31. $-9.3 \cdot (-5.1)$

32. $-95.2 \cdot (-0.12)$

33. $(-0.4)^3$

ERROR ANALYSIS Describe and correct the error.
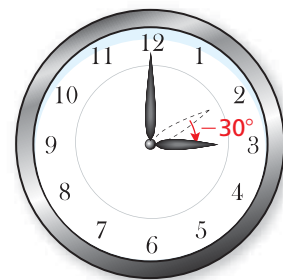
34.

✗ $-2.2 \times 3.7 = 8.14$

35.

✗ $-\dfrac{1}{4} \div \dfrac{3}{2} = -\dfrac{4}{1} \times \dfrac{3}{2} = -\dfrac{12}{2} = -6$

36. **HOUR HAND** The hour hand of a clock moves $-30°$ every hour. How many degrees does it move in $2\dfrac{1}{5}$ hours?

37. **SUNFLOWER SEEDS** How many 0.75-pound packages can be made with 6 pounds of sunflower seeds?

Evaluate.

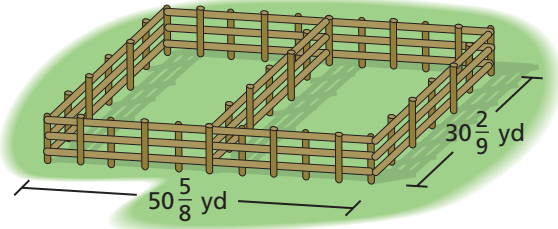**38.** $-4.2 + 8.1 \times (-1.9)$

39. $2.85 - 6.2 \div 2^2$

40. $-3.64 \cdot \left| -5.3 \right| - 1.5^3$

41. $1\frac{5}{9} \div \left(-\frac{2}{3} \right) + \left(-2\frac{3}{5} \right)$

42. $-3\frac{3}{4} \times \frac{5}{6} - 2\frac{1}{3}$

43. $\left(-\frac{2}{3} \right)^2 - \frac{3}{4}\left(2\frac{1}{3} \right)$

44. OPEN-ENDED Write two fractions whose product is $-\dfrac{3}{5}$.

$30\frac{2}{9}$ yd

$50\frac{5}{8}$ yd

45. FENCING A farmer needs to enclose two adjacent rectangular pastures. How much fencing does the farmer need?

46. GASOLINE A 14.5-gallon gasoline tank is $\dfrac{3}{4}$ full. How many gallons will it take to fill the tank?

47. BOARDWALK A section of a boardwalk is made using 15 boards. Each board is $9\frac{1}{4}$ inches wide. The total width of the section is 144 inches. The spacing between each board is equal. What is the width of the spacing between each board?

48. RUNNING The table shows the changes in the times (in seconds) of four teammates. What is the mean change?

Teammate	Change
1	-2.43
2	-1.85
3	0.61
4	-1.45

49. **Critical Thinking** Consider $(-2)^1$, $(-2)^2$, $(-2)^3$, $(-2)^4$, $(-2)^5$, and $(-2)^6$.

    a. Evaluate each expression.

    b. What pattern do you notice?

    c. What is the sign of $(-2)^{49}$?

Fair Game Review What you learned in previous grades & lessons

Add or subtract.

50. $-6.2 + 4.7$

51. $-8.1 - (-2.7)$

52. $\dfrac{9}{5} - \left(-2\dfrac{7}{10} \right)$

53. $-4\dfrac{5}{6} + \left(-3\dfrac{4}{9} \right)$

54. MULTIPLE CHOICE What are the coordinates of the point in quadrant IV?

    (A) $(-4, 1)$

    (B) $(-3, -3)$

    (C) $(0, -2)$

    (D) $(3, -3)$