

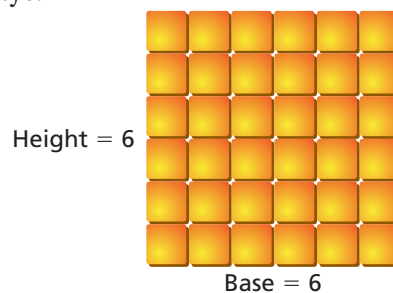
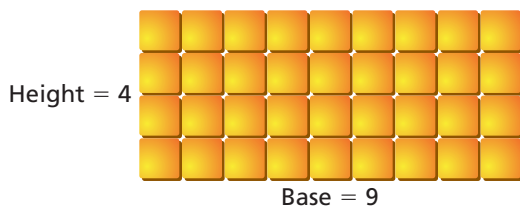
## 3.8 Inverse Variation

**Essential Question** How can you recognize when two variables are inversely proportional?

### 1 ACTIVITY: Comparing the Height and the Base

Work with a partner.

- a. There are nine ways to arrange 36 square blocks to form a rectangle. Here are two ways. Find the other seven ways.



- b. Order the nine ways according to height. Record your results in a table.

Height, $h$	Base, $b$	Area, $A$
4	9	$A = 9 \cdot 4 = 36$
6	6	$A = 6 \cdot 6 = 36$

- c. Look at the first and second columns. Complete each sentence.
- When the height increases, the base .
  - When the height decreases, the base .

In Activity 1, the relationship between the height and the base is an example of **inverse variation**. You can describe the relationship with an equation.

$$h = \frac{36}{b} \quad h \text{ and } b \text{ are inversely proportional.}$$

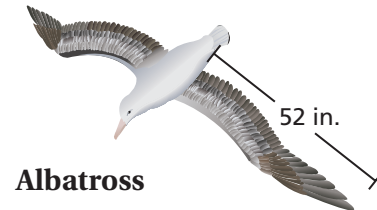
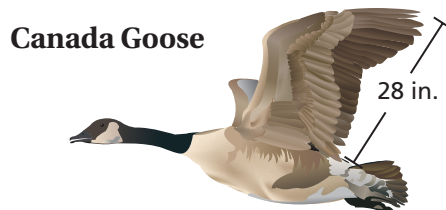
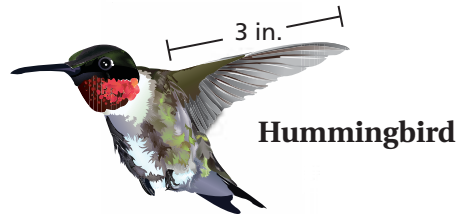
## 2 ACTIVITY: Comparing Direct and Inverse Variation

Work with a partner. Discuss each description. Tell whether the two variables are examples of *direct variation* or *inverse variation*. Use a table to explain your reasoning. Write an equation that relates the variables.

- You bring 200 cookies to a party. Let  $n$  represent the number of people at the party and  $c$  represent the number of cookies each person receives.
- You work at a restaurant for 20 hours. Let  $r$  represent your hourly pay rate and  $p$  represent the total amount you earn.
- You are going on a 240-mile trip. Let  $t$  represent the number of hours driving and  $s$  represent the speed of the car.

### What Is Your Answer?

- IN YOUR OWN WORDS** How can you recognize when two variables are inversely proportional? Explain how a table can help you recognize inverse variation.
- SCIENCE** The *wing beat frequency* of a bird is the number of times per second the bird flaps its wings.



Which of the following seems true? Explain your reasoning.

- Wing length and wing beat frequency are directly proportional.
  - Wing length and wing beat frequency are inversely proportional.
  - Wing length and wing beat frequency are unrelated.
- SCIENCE** Think of an example in science where two variables are inversely proportional.

### Practice

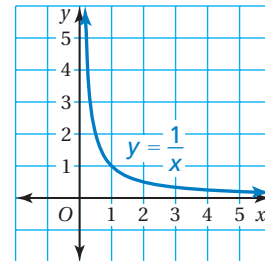
Use what you learned about inverse variation to complete Exercises 4–7 on page 146.

**Key Vocabulary** 

 inverse variation,  
p. 144

 **Key Idea**
**Inverse Variation**
**Words**

Two quantities  $x$  and  $y$  show **inverse variation** when  $y = \frac{k}{x}$ , where  $k$  is a number and  $k \neq 0$ .

**Graph**

**EXAMPLE 1** Identifying Direct and Inverse Variation

**Study Tip**

Other ways to say that  $x$  and  $y$  show inverse variation are “ $y$  varies inversely with  $x$ ” and “ $x$  and  $y$  are inversely proportional.”

Tell whether  $x$  and  $y$  show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

a.  $5y = x$

$$y = \frac{1}{5}x \quad \text{Solve for } y.$$

- ❖ The equation can be written as  $y = kx$ . So,  $x$  and  $y$  show direct variation.

b.  $\frac{1}{3}y = \frac{1}{x}$

$$y = \frac{3}{x} \quad \text{Solve for } y.$$

- ❖ The equation can be written as  $y = \frac{k}{x}$ . So,  $x$  and  $y$  show inverse variation.

c.  $-x = y + 7$

$$-x - 7 = y \quad \text{Solve for } y.$$

- ❖ The equation cannot be written as  $y = kx$  or  $y = \frac{k}{x}$ . So,  $x$  and  $y$  do *not* show direct or inverse variation.

**On Your Own**

Tell whether  $x$  and  $y$  show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

1.  $y - 1 = 2x$

2.  $\frac{1}{5}y = x$

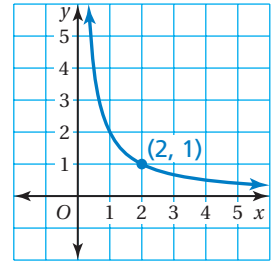
3.  $2y = \frac{1}{x}$

**Now You're Ready**  
Exercises 4–15

## EXAMPLE 2 Standardized Test Practice

In the graph,  $x$  and  $y$  show inverse variation. Which equation relates  $x$  and  $y$ ?

- (A)  $y = -\frac{2}{x}$       (B)  $y = \frac{2}{x}$   
 (C)  $y = -2x$       (D)  $y = 2x$



The graph passes through  $(2, 1)$ . Substitute to find  $k$ .

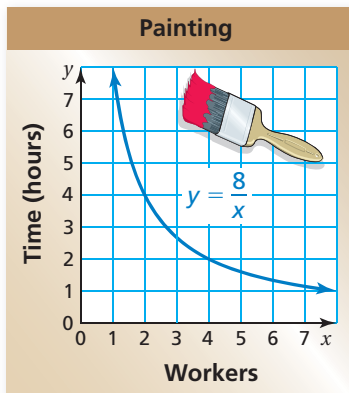
$$y = \frac{k}{x} \quad \text{Write inverse variation equation.}$$

$$1 = \frac{k}{2} \quad \text{Substitute 2 for } x \text{ and 1 for } y.$$

$$2 = k \quad \text{Solve for } k.$$

∴ So, the equation  $y = \frac{2}{x}$  relates  $x$  and  $y$ . The correct answer is (B).

## EXAMPLE 3 Real-Life Application



The graph shows the number of hours  $y$  it takes  $x$  workers to paint a room. (a) How does  $y$  change as  $x$  increases? (b) Do  $x$  and  $y$  show direct or inverse variation? (c) How many hours does it take five workers to paint the room?

- From the graph, you can see that  $y$  decreases as  $x$  increases. So, as the number of workers increases, the time to paint the room decreases.
- The equation is written as  $y = \frac{k}{x}$ . So,  $x$  and  $y$  show inverse variation.
- Use the equation to find  $y$  when  $x = 5$ .

$$y = \frac{8}{x} \quad \text{Write equation.}$$

$$= \frac{8}{5} = 1.6 \quad \text{Substitute. Then simplify.}$$

∴ It takes 1.6 hours for five workers to paint the room.

### On Your Own

**Now You're Ready**  
Exercises 23 and 24

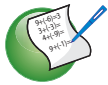
- Suppose  $y$  varies inversely with  $x$  and  $y = 3$  when  $x = 1$ . Write an equation that relates  $x$  and  $y$ .
- WHAT IF?** In Example 3, how many hours does it take three workers to paint the room?

# 3.8 Exercises



## Vocabulary and Concept Check

- WRITING** What does it mean for  $x$  and  $y$  to vary inversely?
- NUMBER SENSE** When  $x$  increases from 1 to 10, does  $\frac{1}{x}$  increase or decrease?
- OPEN-ENDED** Describe a real-life situation that shows inverse variation.



## Practice and Problem Solving

Tell whether  $x$  and  $y$  show *direct variation*, *inverse variation*, or *neither*. Explain your reasoning.

- $y = \frac{1}{x}$
  - $xy = 8$
  - $y - x = 0$
  - $\frac{1}{2}y = 2x$
  - $\frac{y}{3} = \frac{2}{x}$
  - $y - 2 = \frac{7}{x}$
  - $x = y + 9$
  - $x = 4y$
  - $y = \frac{5}{2x}$
  - $2y = \frac{6}{x}$
  - $\frac{5x}{3} = \frac{y}{4}$
  - $x = \frac{7 + y}{2}$

- ERROR ANALYSIS** Describe and correct the error in telling whether  $x$  and  $y$  show inverse variation.



$$\frac{y}{2} = \frac{8}{x}$$

The equation does not show inverse variation because it is not of the form  $y = \frac{k}{x}$ .

Graph the data. Tell whether  $x$  and  $y$  show *direct variation* or *inverse variation*.

17.

$x$	-2	2	4	6
$y$	-1	1	2	3

18.

$x$	0.5	1	3	6
$y$	6	3	1	0.5

19.

$x$	2	5	8	20
$y$	10	4	2.5	1

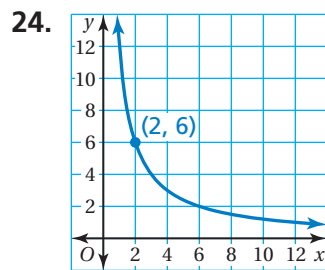
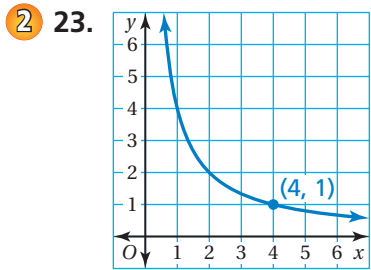
20.

$x$	2	4	8	11
$y$	1.5	3	6	8.25

Tell whether  $x$  and  $y$  show *direct variation* or *inverse variation*. Explain.

- STADIUM** The time  $y$  it takes to empty a stadium and the number  $x$  of open exits are related by the equation  $y = \frac{0.8}{x}$ .
- TRAVEL** The number  $y$  of miles driven and the number  $x$  of gallons of gas used are related by the equation  $y = 28.5x$ .

The variables  $x$  and  $y$  vary inversely. Write an equation relating  $x$  and  $y$ .



25. **BICYCLING** The table shows the times it takes to bicycle 12 miles at various speeds.

Speed (mi/h)	12	6	3	2
Time (h)	1	2	4	6

- Does the time  $t$  vary inversely with the speed  $s$ ? If so, write an equation relating  $t$  and  $s$ .
- What time corresponds to a speed of 4 miles per hour?



26. **MARTIAL ARTS** It takes 3.6 pounds of force to break a 5-foot board.

- You remember from science that force and board length vary directly or inversely, but you've forgotten which. How can you use reason to remember?
  - How much force does it take to break the board shown?
27. **SALARY** A salesperson has a fixed weekly salary. The person works twice as many hours this week as last week. What happens to the person's hourly rate?

28. **Reasoning** The price per person to rent a limousine varies inversely with the number of passengers. It costs \$90 each for five people. How many people are renting the limousine when the cost per person is \$56.25?



## Fair Game Review What you learned in previous grades & lessons

Find the percent of the number. *(Skills Review Handbook)*

29. 40% of 220      30. 32% of 275      31. 84% of 75      32. 21% of 300

Tell whether the ratios form a proportion. *(Section 3.3)*

33.  $\frac{9}{15}, \frac{18}{30}$       34.  $\frac{21}{9}, \frac{18}{8}$       35.  $\frac{42}{91}, \frac{24}{52}$       36.  $\frac{24}{38}, \frac{36}{57}$

37. **MULTIPLE CHOICE** A gumball machine contains 1000 gumballs. The ratio of red gumballs to the total number of gumballs is 1 : 4. How many red gumballs are in the machine? *(Section 3.5)*

- (A) 150      (B) 250      (C) 400      (D) 750