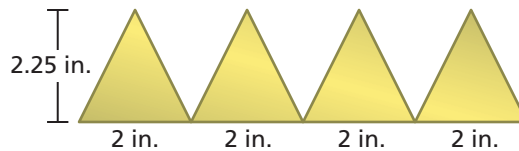**Essential Question** How can you find the volume of a pyramid?

1 ACTIVITY: Finding a Formula Experimentally

Work with a partner.

- **Draw the two nets on cardboard and cut them out.**

2.25 in.

2 in.   2 in.   2 in.   2 in.

2 in.   2 in.   2 in.

2 in.

2 in.

2 in.

2 in.

- **Fold and tape the nets to form an open square box and an open pyramid.**

- **Both figures should have the same size square base and the same height.**

- **Fill the pyramid with pebbles. Then pour the pebbles into the box. Repeat this until the box is full. How many pyramids does it take to fill the box?**

- **Use your result to find a formula for the volume of a pyramid.**
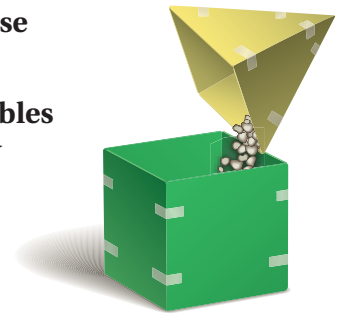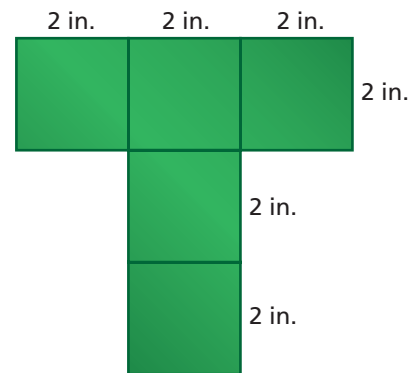
2 ACTIVITY: Comparing Volumes

Work with a partner. You are an archeologist studying two ancient pyramids. What factors would affect how long it took to build each pyramid? Given similar conditions, which pyramid took longer to build? Explain your reasoning.

Cholula Pyramid in Mexico
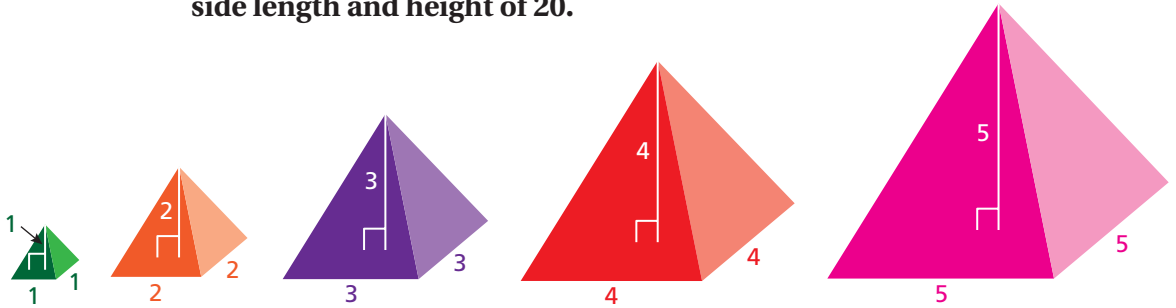Height: about 217 ft
Base: about 1476 ft by 1476 ft

Cheops Pyramid in Egypt
Height: about 480 ft
Base: about 755 ft by 755 ft

3 ACTIVITY: Finding and Using a Pattern

Work with a partner.

- Find the volumes of the pyramids.
- Organize your results in a table.
- Describe the pattern.
- Use your pattern to find the volume of a pyramid with a side length and height of 20.

4 ACTIVITY: Breaking a Prism into Pyramids

Work with a partner. The rectangular prism can be cut to form three pyramids. Show that the sum of the volumes of the three pyramids is equal to the volume of the prism.

a.

b.

c.
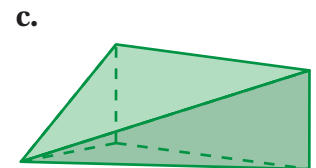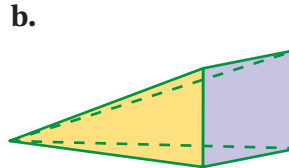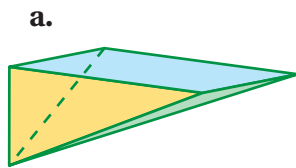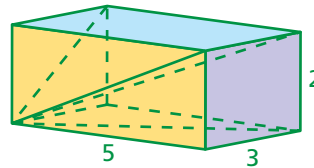
What Is Your Answer?

5. **IN YOUR OWN WORDS** How can you find the volume of a pyramid?

6. Write a general formula for the volume of a pyramid.

Practice

Use what you learned about the volumes of pyramids to complete Exercises 4–6 on page 314.
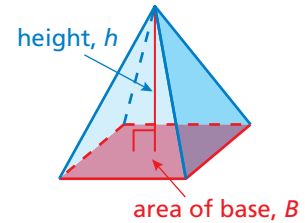
🔑 Key Idea

Volume of a Pyramid

Words The volume V of a pyramid is one-third the product of the area of the base and the height of the pyramid.

height, h

Area of base

Algebra $V = \dfrac{1}{3}Bh$

Height of pyramid
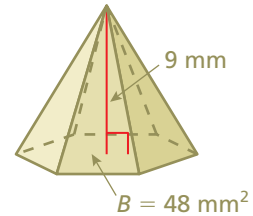
area of base, B

EXAMPLE 1 Finding the Volume of a Pyramid

Find the volume of the pyramid.

$$V = \frac{1}{3}Bh \qquad \text{Write formula for volume.}$$

$$= \frac{1}{3}(48)(9) \qquad \text{Substitute.}$$

$$= 144 \qquad \text{Multiply.}$$

9 mm

$B = 48$ mm^2

∴ The volume is 144 cubic millimeters.

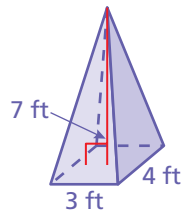---

EXAMPLE 2 Finding the Volume of a Pyramid

Find the volume of the pyramid.

Study Tip

The area of the base of a rectangular pyramid is the product of the length ℓ and the width w.

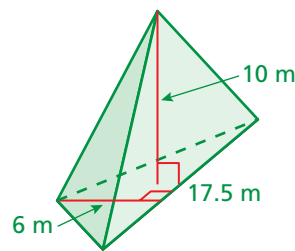You can use $V = \dfrac{1}{3}\ell wh$ to find the volume of a rectangular pyramid.

a.

7 ft

4 ft

3 ft

$$V = \frac{1}{3}Bh$$

$$= \frac{1}{3}(3)(4)(7)$$

$$= 28$$

∴ The volume is 28 cubic feet.

b.

10 m

17.5 m

6 m

$$V = \frac{1}{3}Bh$$
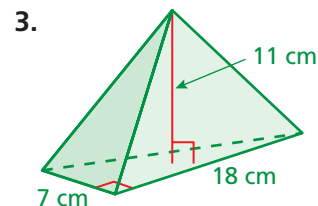
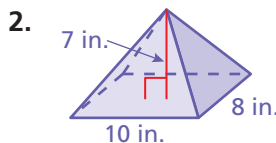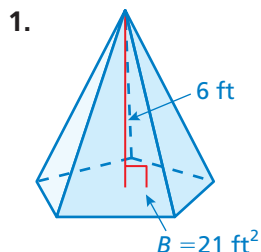$$= \frac{1}{3}\left(\frac{1}{2}\right)(17.5)(6)(10)$$
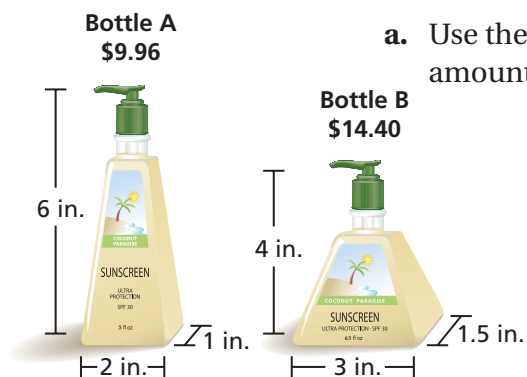
$$= 175$$

∴ The volume is 175 cubic meters.

Find the volume of the pyramid.

1.

6 ft

$B = 21$ ft^2

2.

7 in.

8 in.

10 in.

3.

11 cm

7 cm

18 cm

EXAMPLE 3 **Real-Life Application**

(a) **How many times more sunscreen is in Bottle B than in Bottle A?**

(b) **Which is the better buy?**

Bottle A
$9.96

6 in.

2 in.

1 in.

Bottle B
$14.40

4 in.

3 in.

1.5 in.

a. Use the formula for the volume of a pyramid to estimate the amount of sunscreen in each bottle.

Bottle A	*Bottle B*
$V = \dfrac{1}{3}Bh$	$V = \dfrac{1}{3}Bh$
$= \dfrac{1}{3}(2)(1)(6)$	$= \dfrac{1}{3}(3)(1.5)(4)$
$= 4$ in.^{3}	$= 6$ in.^{3}

So, Bottle B has $\dfrac{6}{4}$, or 1.5 times more sunscreen than Bottle A.

b. Find the unit cost for each bottle.

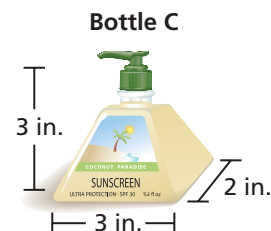| *Bottle A*	*Bottle B*
$\dfrac{\text{cost}}{\text{volume}} = \dfrac{\$9.96}{4 \text{ in.}^3}$	$\dfrac{\text{cost}}{\text{volume}} = \dfrac{\$14.40}{6 \text{ in.}^3}$
$= \dfrac{\$2.49}{1 \text{ in.}^3}$	$= \dfrac{\$2.40}{1 \text{ in.}^3}$

The unit cost of Bottle B is less than the unit cost of Bottle A. So, Bottle B is the better buy.

On Your Own

4. Bottle C is on sale for $13.20. Is Bottle C a better buy than Bottle B in Example 3? Explain.

Bottle C
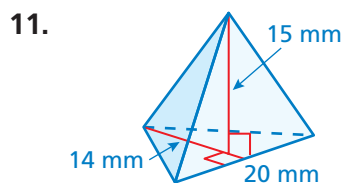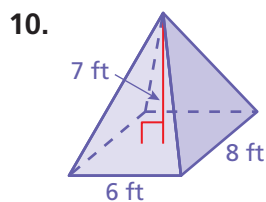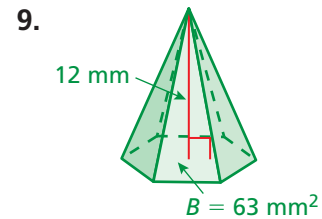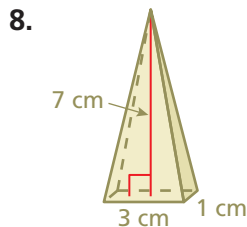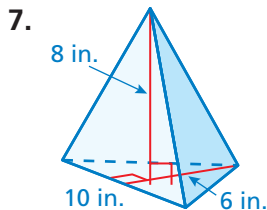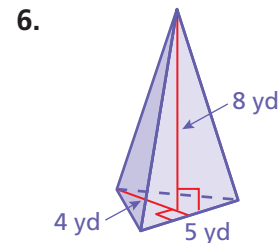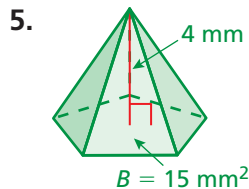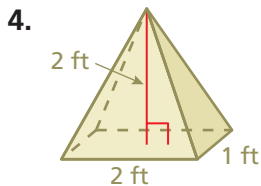
3 in.

3 in.

2 in.

✓ Vocabulary and Concept Check

1. **WRITING** How is the formula for the volume of a pyramid different from the formula for the volume of a prism?

2. **OPEN-ENDED** Describe a real-life situation that involves finding the volume of a pyramid.

3. **REASONING** A triangular pyramid and a triangular prism have the same base and height. How many times greater is the volume of the prism than the volume of the pyramid?

Practice and Problem Solving

Find the volume of the pyramid.

4.

2 ft

1 ft

2 ft

5.

4 mm

$B = 15$ mm²

6.

8 yd

4 yd   5 yd

7.

8 in.

10 in.   6 in.

8.

7 cm

3 cm   1 cm

9.

12 mm

$B = 63$ mm²

10.

7 ft

8 ft

6 ft

11.

15 mm

14 mm   20 mm

12. **PARACHUTE** In 1483, Leonardo da Vinci designed a parachute. It is believed that this was the first parachute ever designed. In a notebook, he wrote "If a man is provided with a length of gummed linen cloth with a length of 12 yards on each side and 12 yards high, he can jump from any great height whatsoever without injury." Find the volume of air inside Leonardo's parachute.
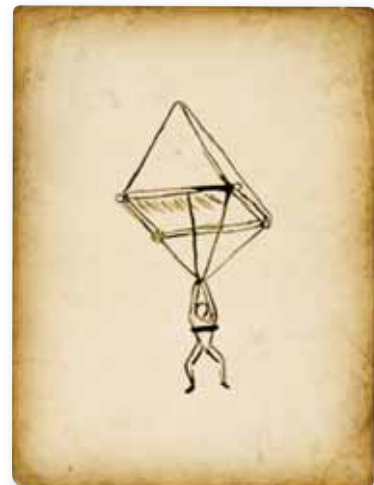
Not drawn to scale

Copy and complete the table to find the area of the base B or the height h of the pyramid.
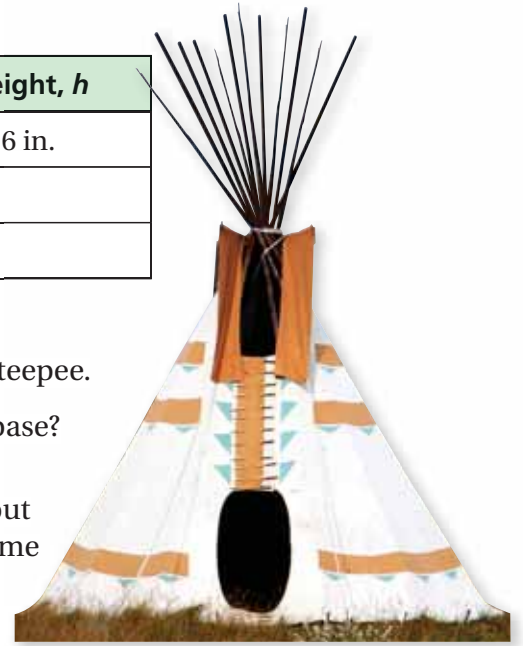
	Volume, V	Area of Base, B	Height, h
13.	60 in.^{3}		6 in.
14.	144 cm^3	48 cm^2	
15.	135 ft^3	54 ft^2	
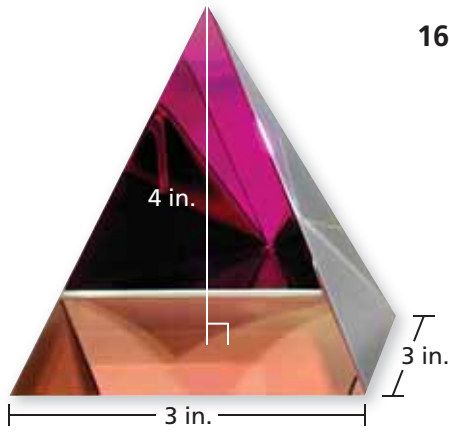
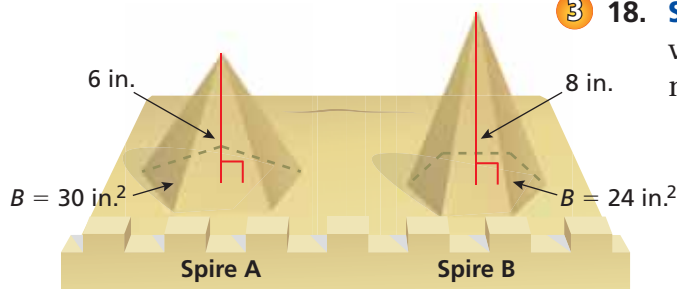**16. TEEPEE** Use the photo of the teepee.

    a. What is the shape of the base? How can you tell?

    b. The teepee's height is about 10 feet. Estimate the volume of the teepee.

4 in.

3 in.

3 in.

Paperweight

17. PAPERWEIGHT How much glass is needed to manufacture 1000 paperweights? Explain your reasoning.

③ 18. SPIRE Which sandcastle spire has a greater volume? How much more sand is required to make the spire with the greater volume?

6 in.

8 in.

$B = 30$ in.^{2}

$B = 24$ in.^{2}

Spire A

Spire B

19. OPEN-ENDED A pyramid has a volume of 40 cubic feet and a height of 6 feet. Find one possible set of dimensions of the rectangular base.

20. Reasoning Do the two solids have the same volume? Explain.

z

y

x

$3z$

y

x
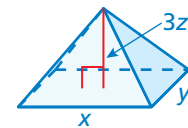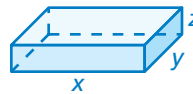
Fair Game Review What you learned in previous grades & lessons

Simplify the expression.

21. $\frac{1}{3} \times 12 \times 7$ **22.** $\frac{1}{3} \times 8 \times 27$ **23.** $\frac{1}{3} \times 6^2 \times 5$ **24.** $\frac{1}{3} \times 2^2 \times 15$

25. MULTIPLE CHOICE You spend 25% of your money on a shirt. Then you spend $\frac{1}{6}$ of the remainder on lunch. Lunch costs $8. What percent of your money is spent on lunch?

    Ⓐ 4.2%   **Ⓑ** 12.5%   **Ⓒ** 16.7%   **Ⓓ** 32%