English Spanish

9.2 Theoretical Probability

Essential Question How can you find a theoretical probability?

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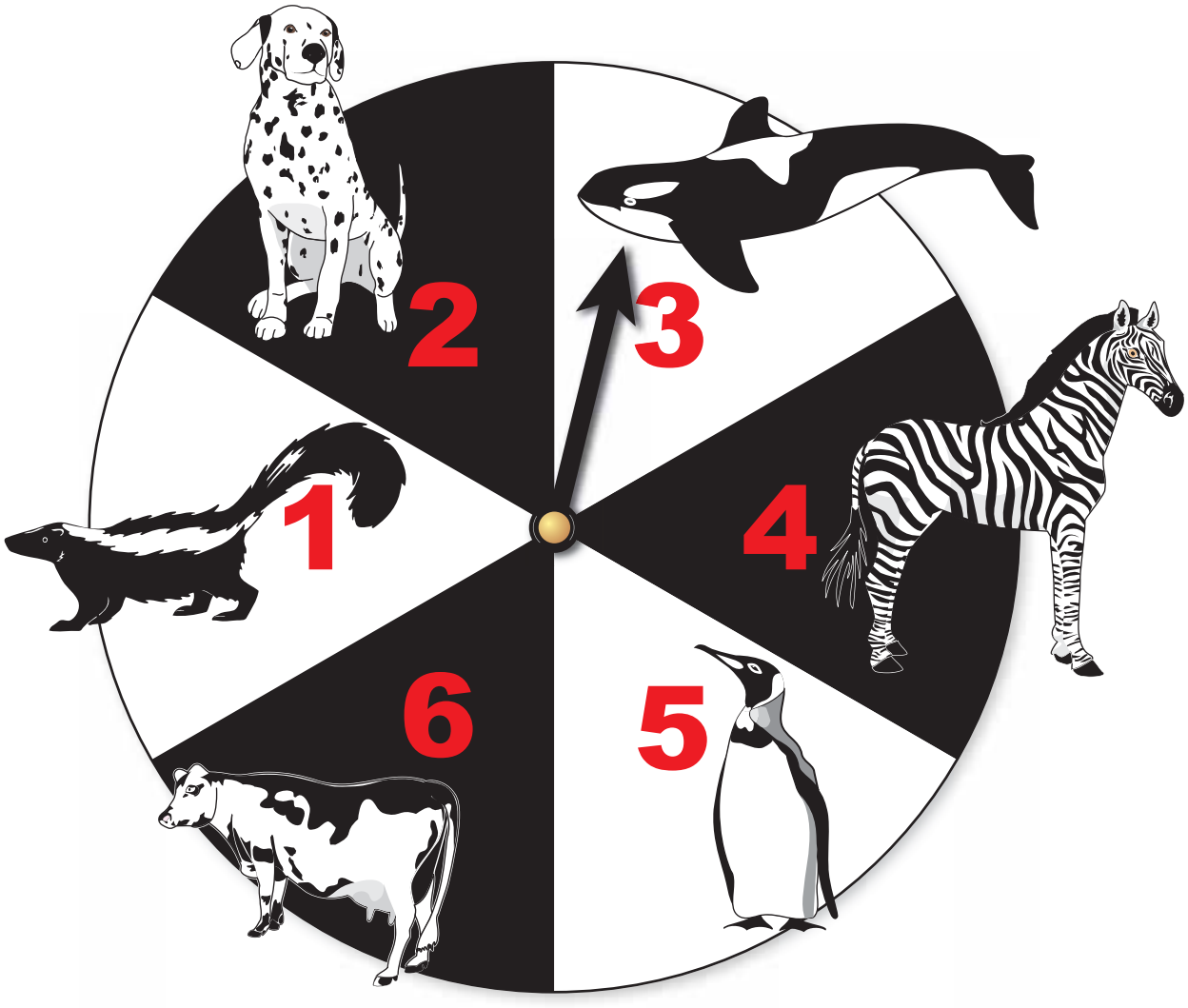
1 ACTIVITY: Black and White Spinner Game

Work with a partner. You work for a game company. You need to create a game that uses the spinner below.

a. Write rules for a game that uses the spinner. Then play it.

b. After playing the game, do you want to revise the rules? Explain.



c. Each pie-shaped section of the spinner is the same size. What is the measure of the central angle of each section?

d. What is the probability that the spinner will land on 1? Explain.

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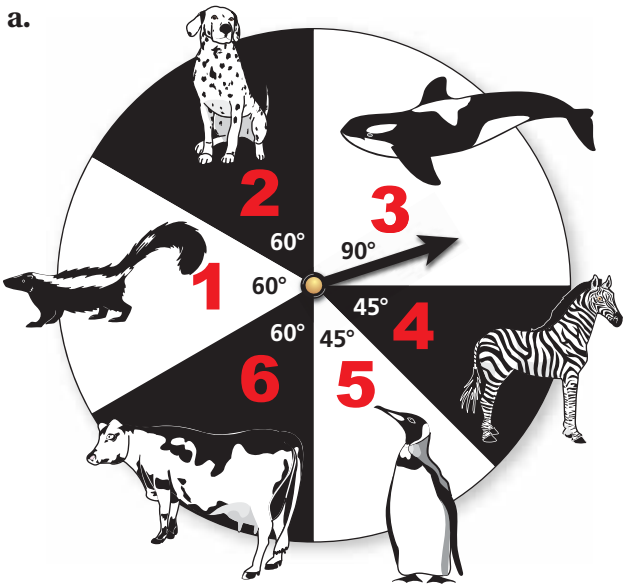
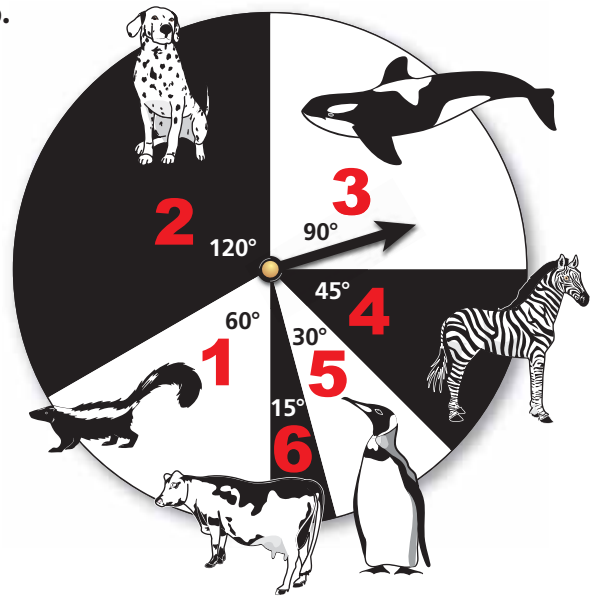
2 ACTIVITY: Changing the Spinner

Work with a partner. For each spinner, find the probability of landing on each number. Do your rules from Activity 1 make sense for these spinners? Explain.

a.



b.



3 ACTIVITY: Is This Game Fair?

Work with a partner. Apply the following rules to each spinner in Activities 1 and 2. Is the game fair? If not, who has the better chance of winning?

- **Take turns spinning the spinner.**
- **If the spinner lands on an odd number, Player 1 wins.**
- **If the spinner lands on an even number, Player 2 wins.**

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find a theoretical probability?
5. Find and describe a career in which probability is used. Explain why probability is used in that career.
6. Two people play the following game.

   Each player has 6 cards numbered 1, 2, 3, 4, 5, and 6. At the same time, each player holds up one card. If the product of the two numbers is odd, Player 1 wins. If the product is even, Player 2 wins. Continue until both players are out of cards. Which player is more likely to win? Why?

Practice Use what you learned about theoretical probability to complete Exercises 4–7 on page 394.

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9.2 Lesson

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Key Vocabulary
theoretical probability, *p. 392*
fair experiment, *p. 393*

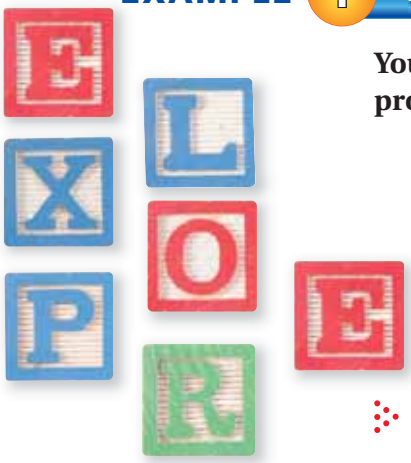
Key Idea

Theoretical Probability

When all possible outcomes are equally likely, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the number of possible outcomes. The probability of an event is written as $P(\text{event})$.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

EXAMPLE 1 Finding a Theoretical Probability



You randomly choose one of the letters shown. What is the theoretical probability of choosing a vowel?

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

$$P(\text{vowel}) = \frac{3}{7}$$

← There are 3 vowels.
← There is a total of 7 letters.

∴ The probability of choosing a vowel is $\frac{3}{7}$ or about 43%.

EXAMPLE 2 Using a Theoretical Probability

The theoretical probability that you randomly choose a green marble from a bag is $\frac{3}{8}$. There are 40 marbles in the bag. How many are green?

$$P(\text{green}) = \frac{\text{number of green marbles}}{\text{total number of marbles}}$$

$$\frac{3}{8} = \frac{n}{40}$$ Substitute. Let n be the number of green marbles.

$$15 = n$$ Multiply each side by 40.

∴ There are 15 green marbles in the bag.

On Your Own

Now You're Ready
Exercises 4–11

1. In Example 1, what is the theoretical probability of choosing an X?
2. The theoretical probability that you spin an odd number on a spinner is 0.6. The spinner has 10 sections. How many sections have odd numbers?

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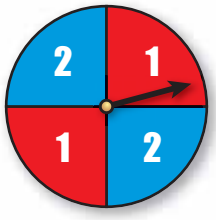
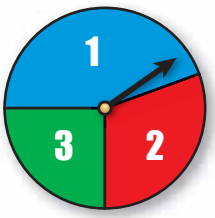
An experiment is **fair** if all of its possible outcomes are equally likely.

Study Tip

A game is fair if every player has the same probability of winning.

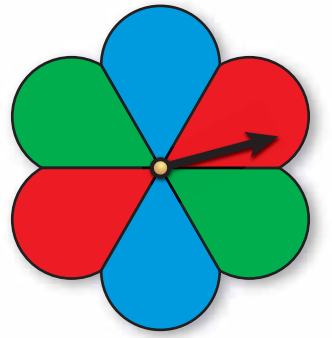


The spinner is equally likely to land on 1 or 2. The spinner is fair.



The spinner is more likely to land on 1 than on either 2 or 3. The spinner is *not* fair.

EXAMPLE 3 Making a Prediction



Scoring Rules:

- You get one point when the spinner lands on blue or green.
- Your friend gets one point when the spinner lands on red.
- The first person to get 10 points wins.

You and your friend play the game. (a) Is the spinner fair? (b) Is the game fair? (c) Predict the number of turns it will take you to win.

a. Yes, the spinner is fair because it is equally likely to land on red, blue, or green.

b. Find and compare the theoretical probabilities of the events.

$$\textbf{You: } P(\text{blue or green}) = \frac{\text{number of blue or green sections}}{\text{total number of sections}}$$

$$= \frac{4}{6} = \frac{2}{3}$$

$$\textbf{Your friend: } P(\text{red}) = \frac{\text{number of red sections}}{\text{total number of sections}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

∴ It is more likely that the spinner will land on blue or green than on red. Because your probability is greater, the game is *not* fair.

c. Write and solve an equation using $P(\text{blue or green})$ found in part (b). Let x be the number of turns it will take you to win.

$\frac{2}{3}x = 10$ Write equation.

$x = 15$ Multiply each side by $\frac{3}{2}$.

∴ So, you can predict that it will take you 15 turns to win.

On Your Own

Now You're Ready
Exercises 12–14

3. WHAT IF? In Example 3, you get one point when the spinner lands on blue or green. Your friend gets one point when the spinner lands on red or blue. The first person to get 5 points wins. Is the game fair? Explain.

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9.2 Exercises

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Vocabulary and Concept Check

1. **VOCABULARY** An event has a theoretical probability of 0.5. What does this mean?
2. **OPEN-ENDED** Describe an event that has a theoretical probability of $\frac{1}{4}$.
3. **WHICH ONE DOESN'T BELONG?** Which spinner does *not* belong with the other three? Explain your reasoning.


Spinner 1


Spinner 2


Spinner 3


Spinner 4

Practice and Problem Solving

Use the spinner to determine the theoretical probability of the event.

1 4. Spinning red
5. Spinning a 1
6. Spinning an odd number
7. Spinning a multiple of 2
8. Spinning a number less than 7
9. Spinning a 7
10. **LETTERS** Each letter of the alphabet is printed on an index card. What is the theoretical probability of randomly choosing any letter except Z?

2 11. **GAME SHOW** On a game show, a contestant randomly chooses a chip from a bag that contains numbers and strikes. The theoretical probability of choosing a strike is $\frac{3}{10}$. There are 30 chips in the bag. How many are strikes?

A number cube is rolled. Determine if the game is fair. If it is *not* fair, who has the greater probability of winning?

3 12. You win if the number is odd. Your friend wins if the number is even.
13. You win if the number is less than 3. If it is not less than 3, your friend wins.
14. **SCORING POINTS** You get one point if a 1 or a 2 is rolled on the number cube. Your friend gets one point if a 5 or a 6 is rolled. The first person to 5 points wins.
    a. Is the number cube fair? Is the game fair? Explain.
    b. Predict the number of turns it will take you to win.

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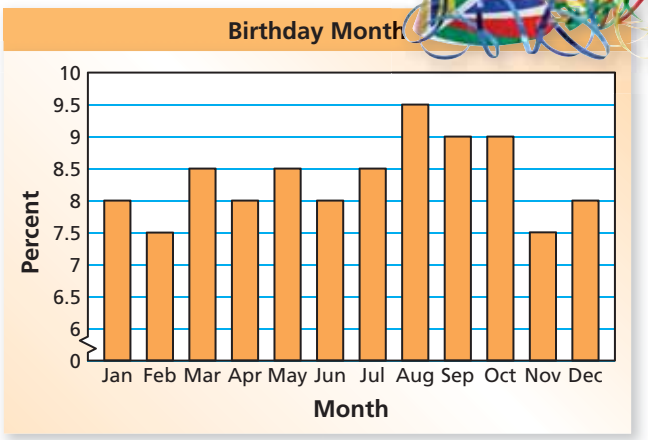
15. HISTORY You write a report about your favorite president. Your friend writes a report on a randomly chosen president. What is the theoretical probability that you write reports on the same president?

16. BIRTHDAYS The bar graph shows the birthday months of all 200 employees at a local business.



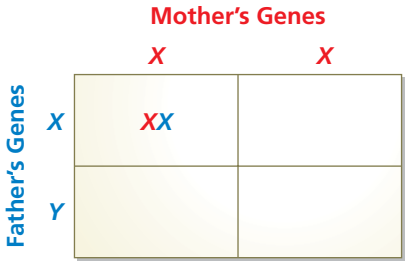
a. What is the theoretical probability of randomly choosing a person at the business who was born in a month with an R in its name?

b. What is the theoretical probability of randomly choosing a person at the business who has a birthday in the first half of the year?

17. SCHEDULING There are 16 females and 20 males in a class.

a. What is the theoretical probability of randomly choosing a female from the class?

b. One week later, there are 45 students in the class. The theoretical probability of randomly selecting a female is the same as last week. How many males joined the class?



A Punnett square is a grid used to show possible gene combinations for the offspring of two parents. In the Punnett square shown, a boy is represented by *XY*. A girl is represented by *XX*.

18. Complete the Punnett square.

19. Explain why the probability of two parents having a boy or having a girl is equally likely.

20. Critical Thinking Two parents each have the gene combination *Cs*. The gene *C* is for curly hair. The gene *s* is for straight hair.

a. Make a Punnett square for the two parents. If all outcomes are equally likely, what is the probability of a child having the gene combination *CC*?

b. Any gene combination that includes a *C* results in curly hair. If all outcomes are equally likely, what is the probability of a child having curly hair?

Fair Game Review What you learned in previous grades & lessons

Multiply. SECTION 2.3

21. $\frac{1}{2} \times \frac{1}{2}$ **22.** $-\frac{1}{6} \times \frac{2}{3}$ **23.** $-\frac{3}{5} \times \frac{7}{8}$ **24.** $\frac{4}{5} \times \frac{1}{36}$

25. MULTIPLE CHOICE What is the mean of the numbers 11, 6, 12, 22, 7, 8, and 4? SKILLS REVIEW HANDBOOK

(A) 4 (B) 8 (C) 10 (D) 70